Three Oscillation Levels

Part 3: The Cosmological Scale (Level 2)

The Universe Wavefunction ψ_{univ} and the Generator r^{17}/v^8

Concluding part of a three-part series. Parts 1 (The Planck Step) and 2 (The Quantum Scale) established the base-15 unit-number framework, the key operator $y = M^2T$, and the electron wavefunction ψ_{e}.

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1. Recap of the Framework

The base-15 geometry assigns to every physical quantity both a *unit number* θ (an integer) and a pair of *scalars* (r, v) that encode dimensional content. The two anchoring scalars are:

- r — units $(\frac{kg\ m}{s})^{1/4}$; unit number $\theta(r) = 8$.
- v — units m/s; unit number $\theta(v) = 17$.

The fundamental quantities relevant to Level 2 are listed in Table 1.

Table 1: Base-15 scalar and unit-number assignments for Level 2 quantities.

Quantity	Symbol	Scalars	θ
Planck temperature	T_{p}	v^4/r^6	20
Sqrt-momentum	P	r^2	16
Mass	M	r^4/v	15
Time	T	r^9/v^6	−30
Key operator	$y = M^2T$	r^{17}/v^8	0

From Part 2, the *key operator* is

$$y \;=\; M^2T \;=\; \pi\,\frac{r^{17}}{v^8}, \qquad \theta(y) = 0, \tag{1}$$

the fundamental increment of the simulation loop `FOR` $t = 1$ `TO` t_{end} : add Planck units; `NEXT`.

2. The Level 2 Universe Clock

The universe radiation clock ends when the CMB temperature reaches absolute zero. From the CMB–Planck-unit scaffolding [1], the clock endpoint is

$$t_{\text{end}} = \left(\frac{T_{\text{p}}}{8\pi}\right)^4 \approx 1.014 \times 10^{123}\, t_{\text{p}}, \tag{2}$$

where t_{p} is the Planck time. At each tick t_{age} the CMB temperature evolves as

$$T_{\text{CMB}}(t_{\text{age}}) = \frac{T_{\text{p}}}{8\pi\sqrt{t_{\text{age}}}}, \tag{3}$$

so that $T_{\text{CMB}} \to 0$ as $t_{\text{age}} \to t_{\text{end}}$.

Expanding the geometric (scalar-free) part of T_{p} using $T_{\text{p}}^{(\text{geom})} = 2^7\pi^3\Omega^5/a$ (Table 9 of [2]), we obtain

$$t_{\text{end}}^{(\text{geom})} = \left(\frac{2^7\pi^3\Omega^5}{8\pi\, a}\right)^4 = \frac{2^{16}\,\pi^8\,\Omega^{20}}{a^4}. \tag{4}$$

Within the geometric decomposition adopted in this framework, the endpoint scales as a^{-4}, i.e., by the *fourth power of the inverse fine-structure constant* ($\alpha = 1/a$):

$$t_{\text{end}} \propto \alpha^4. \tag{5}$$

3. The Level 2 Dimensional Ratio: P^5/T_{p}^4

We now derive the central scalar combination of Level 2 by combining the fourth power of T_{p} and the fifth power of P.

3.1. Individual powers

$$T_{\text{p}}^4 : \quad \text{scalars} = \left(v^4/r^6\right)^4 = v^{16}/r^{24}, \qquad \theta = 4 \times 20 = 80, \tag{6}$$

$$P^5 : \quad \text{scalars} = \left(r^2\right)^5 = r^{10}, \qquad \theta = 5 \times 16 = 80. \tag{7}$$

3.2. The ratio

$$\frac{P^5}{T_{\text{p}}^4} : \quad \text{scalars} = \frac{r^{10}}{v^{16}/r^{24}} = \frac{r^{34}}{v^{16}}, \qquad \theta = 80 - 80 = 0. \tag{8}$$

3.3. Equivalence to M^4T^2

The scalar combination r^{34}/v^{16} is reproduced exactly by M^4T^2:

$$M^4T^2: \quad \text{scalars} = \left(\frac{r^4}{v}\right)^4 \cdot \left(\frac{r^9}{v^6}\right)^2 = \frac{r^{16}}{v^4} \cdot \frac{r^{18}}{v^{12}} = \frac{r^{34}}{v^{16}}, \qquad \theta = 4(15) + 2(-30) = 0. \quad (9)$$

Level 2 central identity:

$$\frac{P^5}{T_\mathrm{p}^4} = M^4T^2 = y^2, \qquad \theta = 0, \quad \text{scalars} = r^{34}/v^{16}. \qquad (10)$$

The cosmological ratio of the fifth power of sqrt-momentum to the fourth power of Planck temperature equals the square of the key operator $y = M^2T$.

3.4. Verification in natural units

Setting $\hbar = c = 1$, the Planck time satisfies $t_\mathrm{P} = 1/m_\mathrm{P}$, so

$$M^4T^2\big|_\text{natural} = m_\mathrm{P}^4 \cdot t_\mathrm{P}^2 = m_\mathrm{P}^4 \cdot \frac{1}{m_\mathrm{P}^2} = m_\mathrm{P}^2. \qquad (11)$$

The cosmological state descriptor therefore carries the weight of the *square of the Planck mass*, or equivalently G^{-1} in natural units.

4. The Universe Wavefunction ψ_univ

4.1. The three-level analogy

Table 2 summarises the wavefunction (i.e.: the proposed Level-2 state function) at each level of the oscillation hierarchy.

Table 2: Wavefunction structure at each oscillation level.

Level	Scale	Domain coupling	Wavefunction core
0	Planck step	single tick $T = \pi$	$T = \pi$
1	Quantum scale	charge $\times$ length vs. time	$\psi_\mathrm{e} \sim (AL)^3/T$
2	Cosmological	matter-momentum vs. temperature	$\psi_\text{univ} \sim M^4T^2 = P^5/T_\mathrm{p}^4$

At Level 1, the electron wavefunction couples the *electromagnetic sector* (three magnetic monopoles AL) against time T, with scalars that cancel to unity:

$$\psi_\mathrm{e} = \frac{\sigma_e^3}{2T}, \qquad \text{scalars} = \frac{(r^3/v^2)^3}{r^9/v^6} = 1. \qquad (12)$$

At Level 2, the universe couples the *matter-momentum sector* P^5 against the *temperature sector* T_p^4.

4.2. The proposed form

Incorporating the endpoint factor $a^{-4} = \alpha^4$ from Eq. (5):

Universe wavefunction (Level 2):

$$\psi_{\text{univ}} \;\sim\; \alpha^4 \cdot M^4 T^2 \;=\; \alpha^4 \cdot \frac{P^5}{T_{\text{p}}^4} \;\sim\; \frac{\alpha^4}{G}, \tag{13}$$

where the final form uses $G^{-1} \sim m_{\text{P}}^2$ in natural units. The unit number $\theta = 0$ throughout; scalars $= r^{34}/v^{16}$.

The power $n = 4$ of α is fixed by the closing condition: the simulation terminates when T_{p}^4 reaches its geometric ceiling, a condition controlled by a^4 through Eq. (4). This mirrors how ψ_{e} involves a^3 from the three-monopole structure $(AL)^3$.

4.3. Comparison of Level 1 and Level 2

$$\psi_{\text{e}} \sim a^3 \cdot (AL)^3/T, \quad \text{scalars} = 1 \quad \text{(static invariant)}, \tag{14}$$

$$\psi_{\text{univ}} \sim a^{-4} \cdot M^4 T^2, \quad \text{scalars} = r^{34}/v^{16} \quad \text{(running state density)}. \tag{15}$$

The oscillation completes at $t_{\text{age}} = t_{\text{end}}$ (i.e.: 1 complete cycle where t_{end} is the frequency of the universe 'wave=state', drawing on the electron formula ψ analogy).

5. The Generator r^{17}/v^8

5.1. The canonical null-unit-number scalar

Since $\theta(r) = 8$ and $\theta(v) = 17$, the condition $\theta = 0$ for a monomial $r^m v^{-n}$ requires $8m = 17n$. The minimal positive-integer solution is

$$m = 17, \quad n = 8, \qquad \Rightarrow \qquad \theta\left(\frac{r^{17}}{v^8}\right) = 8(17) - 17(8) = 0. \tag{16}$$

This is the *cross-product* of the unit numbers: each scalar is raised to the unit number of the other. The combination r^{17}/v^8 is therefore the unique minimal $\theta = 0$ monomial in the (r, v) basis.

Generator property. Every $\theta = 0$ scalar combination of r and v is an integer power of this generator:

$$\text{all } \theta = 0 \text{ scalars} = \left\{ \left(\frac{r^{17}}{v^8}\right)^n \;\middle|\; n \in \mathbb{Z} \right\}. \tag{17}$$

5.2. Identity with the key operator

From the definition of y in Eq. (1):

$$y = M^2T = \pi \frac{r^{17}}{v^8}, \tag{18}$$

so r^{17}/v^8 is the *scalar backbone* of the key operator, differing only by the geometric factor π. In SI units, r^{17}/v^8 carries units $\mathrm{kg}^2\,\mathrm{s}$ (mass-squared $\times$ time).

5.3. Squaring gives the Level 2 state

The cosmological state descriptor follows by squaring the generator:

$$\left(\frac{r^{17}}{v^8}\right)^2 = \frac{r^{34}}{v^{16}} = \text{scalars of } M^4T^2 = \psi_{\text{univ}} \text{ core.} \tag{19}$$

The three-level progression of scalar powers is thus:

$$n = 1: \quad r^{17}/v^8 = y/\pi \quad \Rightarrow \quad \text{simulation increment,} \tag{20}$$

$$n = 2: \quad r^{34}/v^{16} = M^4T^2/\pi^2 \quad \Rightarrow \quad \psi_{\text{univ}} \text{ core (Level 2).} \tag{21}$$

5.4. Numerical span and the simulation range

As tabulated in §5.1 of [2], the value of r^{17}/v^8 spans the physical constants over a range of approximately 10^{120}, which maps directly onto the temporal extent of the universe clock:

$$\frac{t_{\text{end}}}{1} = \left(\frac{T_{\text{p}}}{8\pi}\right)^4 \approx 10^{123}. \tag{22}$$

The generator r^{17}/v^8 is the ruler against which t_{age} is measured: at each tick, one unit of $y = \pi\, r^{17}/v^8$ is added to the simulation state.

5.5. Bridge between the two SI anchors

Finally, the physical interpretation of r^{17}/v^8 as a bridge is summarised as follows:

r (from μ_0)	$\longleftrightarrow$	v (from c)
electromagnetic anchor		kinematic anchor
$\theta(r) = 8$		$\theta(v) = 17$
Cross-product: $r^{\theta(v)}/v^{\theta(r)} = r^{17}/v^8, \quad \theta = 0$		

The combination r^{17}/v^8 is the unique object that simultaneously spans both foundational sectors of the SI system—electromagnetism and kinematics—while remaining unit-number neutral. It is the seam between the two.

6. The Primacy of M and T: The Two-Generator Theorem

A structural observation running throughout the foregoing analysis has remained implicit. We now make it explicit: M and T are the *only* primary quantities in the framework. Every other physical unit—P, L, V, A, T_p—is an algebraic consequence. P is introduced not as a primitive but solely as the vehicle through which the geometric coupling Ω enters the system. Once Ω is present, all remaining SI dimensions are generated automatically.

6.1. The Two Primary Quantities

At Level 0 the Planck step resolves to exactly two primitive objects:

$$M = 1, \qquad \text{scalar} = \frac{r^4}{v}, \qquad \theta = 15, \tag{23}$$

$$T = \pi, \qquad \text{scalar} = \frac{r^9}{v^6}, \qquad \theta = -30. \tag{24}$$

M is the Planck mass: the matter anchor from which gravitational and inertial phenomena derive. $T = \pi$ is the Planck time: its value is π (not 1), connecting the fundamental clock tick to circular geometry—one tick encodes one half-rotation in phase space, the minimal oscillation of the simulation loop. Nothing else exists at Level 0: no length, no charge, no temperature. All richer structure is derived.

6.2. Deriving P from M and T

Sqrt-momentum P is introduced to bring Ω into the algebra. It satisfies

$$P \;=\; \Omega \cdot M^{4/5} \cdot \left(\frac{\pi}{T}\right)^{2/15}. \tag{25}$$

Scalar verification.

$$\begin{aligned} M^{4/5}\,\text{scalar} &= \left(\tfrac{r^4}{v}\right)^{4/5} = \frac{r^{16/5}}{v^{4/5}}, \\ \left(\pi/T\right)^{2/15}\text{scalar} &= \left(\tfrac{v^6}{r^9}\right)^{2/15} = \frac{v^{4/5}}{r^{6/5}}, \\ \text{combined}: \quad & r^{16/5-6/5} \cdot v^{4/5-4/5} = r^{10/5} = r^2. \quad \checkmark \end{aligned} \tag{26}$$

Unit-number verification.

$$\theta(P) = \tfrac{4}{5}(15) + \tfrac{2}{15}(30) + 0 = 12 + 4 = 16. \quad \checkmark \tag{27}$$

Ω carries no scalar content and contributes nothing to θ; it is the sole genuinely new ingredient that P introduces. Clearing the denominator 15 yields the integer-lattice form:

$$\boxed{P^{15} = \Omega^{15} \cdot M^{12} \cdot \frac{\pi^2}{T^2}.} \tag{28}$$

The exponent 15 is exactly the base of the system: fifteen copies of P form one integer-lattice combination of M and T.

6.3. The Universal Derivation Formula

The structure of Eq. (25) is not special to P. For any physical quantity Q whose scalar content is $r^p\, v^q$, the decomposition into primary units follows a single formula. The conditions $4a+9b=p$ and $a+6b=-q$ (matching scalar powers of r and v separately) admit the unique solution

Universal M–T decomposition. For Q with scalar $r^p\, v^q$:

$$Q = \Omega^k \cdot \pi^j \cdot M^a \cdot T^b, \qquad a = \frac{2p+3q}{5}, \quad b = -\frac{p+4q}{15}. \tag{29}$$

The unit-number constraint is automatic: $15a - 30b = 8p + 17q = \theta(Q)$.

Table 3 applies this formula to the five most important quantities.

Table 3: Universal decomposition of derived SI quantities into M^aT^b. "LCD" is the least common denominator of the exponents; the integer form clears it.

Quantity (θ, scalar)	a	b	LCD	Integer form
P ($\theta=16$, r^2)	$\frac{4}{5}$	$-\frac{2}{15}$	15	$P^{15} \propto M^{12}\, T^{-2}$
V ($\theta=17$, v)	$\frac{3}{5}$	$-\frac{4}{15}$	15	$V^{15} \propto M^{9}\, T^{-4}$
L ($\theta=-13$, r^9v^{-5})	$\frac{3}{5}$	$\frac{11}{15}$	15	$L^{15} \propto M^{9}\, T^{11}$
T_{p} ($\theta=20$, v^4r^{-6})	0	$-\frac{2}{3}$	3	$T_{\mathrm{p}}^3 \propto T^{-2}$
$y = M^2T$ ($\theta=0$, $r^{17}v^{-8}$)	2	1	**1**	$y = M^2T$ *(already integer)*

6.4. The Degree-15 Algebraic Extension

The denominators in Eq. (29) divide 5 and 15 respectively, giving least common denominator 15 for the generic case. Raised to the 15th power, every derived quantity lands on the integer M–T lattice:

$$Q^{15} = \Omega^{15k} \cdot \pi^{15j} \cdot M^{\,3(2p+3q)} \cdot T^{-(p+4q)}. \tag{30}$$

The system is a *degree-15 algebraic extension* of the integer M–T lattice. The label "base 15" is not a notational choice: it is the exact degree of this extension, forced by $\theta(M) = 15$ and $\theta(T) = -30$.

Two exceptional cases narrow the period below 15:

T_p: mass-free, period 3. The Planck temperature has $a = 0$ (no M content) and $b = -2/3$, giving LCD $= 3$. More generally, for any quantity Q with $a = 0$ (mass-free), the formula reduces to

$$b = \frac{\theta(Q)}{\theta(T)}, \tag{31}$$

meaning the T-exponent is the *unit-number ratio* of Q to T. For T_p: $b = 20/(-30) = -2/3$. The Planck temperature is a purely temporal object — it carries no mass content and evolves as $T_\mathrm{p} \propto T^{-2/3}$. This is the primary signal of the thermal dominance in Level 2: the universe's temperature history depends only on the time axis of the M–T plane.

$y = M^2T$: period 1 (the unique integer point). The key operator $y = M^2T$ has $a = 2$ and $b = 1$, both pure integers. It is the *only non-trivial* $\theta = 0$ combination of M and T that sits directly on the integer lattice without the degree-15 extension. Every other $\theta = 0$ scalar requires fractional powers of M and T; only y is exact. This is the algebraic reason y is the key operator: it is the *unique integer-lattice $\theta = 0$ object.*

Combined with Eq. (18), this closes the loop:

$$r^{17}/v^8 \;=\; y/\pi \;=\; M^2T/\pi. \tag{32}$$

The generator of all $\theta = 0$ scalar states and the key simulation operator are one and the same object, differing only by the geometric factor π.

6.5. Mathematical Implications

1. **Five SI dimensions from two generators plus one coupling.** The apparent independence of M, L, T_SI, A, K in the SI system is illusory. All five reduce to $\{M, T\}$ via Eq. (29) once Ω is supplied. The SI system is a degree-15 cover of a two-dimensional lattice.

2. **P is the minimal extension element.** The formula $P = \Omega \cdot M^{4/5}(\pi/T)^{2/15}$ cannot yield Ω from M and T alone—Ω is genuinely new. P is the minimal object that "injects" the electromagnetic coupling into the purely gravitational $\{M, T\}$ pair.

3. **The generator has unit period.** Because $y = M^2T$ carries integer exponents, it requires no extension. Every $\theta = 0$ state is an integer power of y (up to geometric factors of π). The entire null scalar lattice is generated by the one object that sits inside the base lattice itself.

4. **T_p decouples from M.** The mass-free property $a = 0$ for T_p explains why the universe clock endpoint $t_\mathrm{end} = (T_\mathrm{p}/8\pi)^4$ is controlled purely by the coupling α^4

(Eq. (5)), with no direct mass dependence. The thermal and gravitational sectors are decoupled at the level of primary units.

5. **Coprimality is preserved by the extension.** Because $\gcd(8, 17) = 1$, the generator $r^{17}/v^8 = M^2T/\pi$ cannot be factored further inside the degree-15 extension. The incommensurability of the electromagnetic and kinematic sectors (Section 5) persists all the way down to the integer M–T lattice.

6.6. Philosophical Implications

Ontological monism. The universe is built from exactly two primitive substances: *matter* (M) and *duration* (T). Length, velocity, charge, temperature—every measurable quantity—is a geometric projection of the M–T pair onto a specific sector of the base-15 lattice. The proliferation of SI base units reflects human measurement convention, not fundamental ontology. At the simulation level, only two state variables exist.

$T = \pi$ and the geometry of time. The Planck time is not the bare unit 1. Its value π is the ratio of a circle's circumference to its diameter: the most primitive measure of rotation. One Planck tick is one half-cycle of the most elementary oscillation. Every derived quantity inherits this circularity—the appearance of π/T and $(\pi/T)^{2/15}$ in the formula for P is the trace of that geometry propagating upward through the degree-15 extension. Duration in this framework is not a line but a winding.

Ω as the sole source of physical richness. Without Ω, the framework collapses to pure M–T geometry: a one-dimensional mass-time manifold with no charge, no length independent of mass and time, no independent temperature. With Ω, the degree-15 extension opens up five dimensions. The fine-structure constant is not a parameter *within* the laws of physics—it is the parameter *of* the laws: the single coupling that elevates a two-variable time series into a physical universe.

Maximal computational economy. In the simulation interpretation, the loop `FOR` $t = 1$ `TO` t_{end} : add y; `NEXT` maintains exactly two running state variables (M and T). No other quantity is stored: P, L, V, A, T_{p} are all computed on demand from M and T via Eq. (29). The entire physical universe is a read-only projection of a two-integer state. This is not merely economical—it is the minimum possible description: two integers, one coupling, one geometric constant (π), fifteen steps to any observable.

7. The M–T Plane: Geometry of the Oscillation Hierarchy

The universal formula (Eq. (29)) assigns to every physical quantity a unique point (a, b) in a two-dimensional real plane with coordinates given by the M-exponent a and the T-exponent b. The entire structure of the oscillation hierarchy is encoded in the geometry of this plane.

7.1. The $\theta = 0$ Line

The unit-number condition $\theta(Q) = 0$ becomes, in (a, b) coordinates:

$$15a - 30b = 0 \quad \Longrightarrow \quad a = 2b. \tag{33}$$

All dimensionless objects in the base-15 sense lie on the line $a = 2b$ through the origin of the M–T plane. The integer lattice points on this line are

$$\{(a, b) = (2n,\, n) : n \in \mathbb{Z}\}, \tag{34}$$

each separated from the next by the vector $(2, 1)$ — exactly the exponents of the key operator $y = M^2T$. The step from one lattice point to the next is multiplication by y.

7.2. σ_e is Mass-Free: Why ψ_e Has Scalars $= 1$

The electron cross-section σ_e has $\theta = -10$ and scalar r^3/v^2 ($p = 3$, $q = -2$). Applying Eq. (29):

$$a = \frac{2(3) + 3(-2)}{5} = \frac{6 - 6}{5} = 0, \qquad b = -\frac{3 + 4(-2)}{15} = -\frac{3 - 8}{15} = \frac{1}{3}. \tag{35}$$

σ_e is *mass-free*: no M content whatsoever. It is a pure cube root of T:

$$\sigma_e \;\propto\; T^{1/3}, \qquad \text{and therefore} \quad \sigma_e^3 \;\propto\; T. \tag{36}$$

This is not a coincidence but a structural necessity. Since $\psi_e = \sigma_e^3/(2T)$, the scalar content of the numerator and denominator are identical:

$$\text{scalars}\big(\sigma_e^3\big) = \big(r^3/v^2\big)^3 = r^9/v^6 = \text{scalars}(T). \tag{37}$$

The electron wavefunction has scalars $= 1$ because σ_e *is* the cube root of T in the M–T decomposition. The cancellation in ψ_e is not incidental; it is the defining feature of a mass-free $\theta = 0$ object at $n = 0$ in the lattice.

This also places σ_e alongside T_p as a mass-free, period-3 object:

Table 4: The two mass-free quantities and their T-exponents. Both live on the $a = 0$ axis and have LCD $= 3$.

Quantity	θ	a	b	LCD	Power law
σ_e (em cross-section)	-10	0	$+\frac{1}{3}$	3	$\sigma_e \propto T^{+1/3}$
T_p (Planck temperature)	$+20$	0	$-\frac{2}{3}$	3	$T_p \propto T^{-2/3}$

Both quantities sit on the vertical axis of the M–T plane ($a = 0$), and both involve cube roots of T. Their unit numbers sum to $-10 + 20 = 10$ and their b-exponents sum to $+1/3 + (-2/3) = -1/3$: their product $\sigma_e T_\mathrm{p} \propto T^{-1/3}$ is again mass-free. The mass-free sector is closed under multiplication.

In the quark model [3], the magnetic monopole σ_e is a D quark and the temperature monopole σ_{Tp} is a U quark.

$$D = \sigma_e, \quad \text{unit} = u^{-10}, \quad \text{charge} = -\frac{e}{3}, \quad \text{scalars} = \frac{r^3}{v^2}$$

$$U = \sigma_t, \quad \text{unit} = u^{20}, \quad \text{charge} = \frac{2e}{3}, \quad \text{scalars} = \frac{v^4}{r^6}$$

7.3. The Two Sectors of the M–T Plane

The M–T plane divides naturally into two orthogonal sectors, corresponding to the two oscillation levels:

Sector	Condition	Inhabitants
Thermal / electromagnetic	$a = 0$ (mass-free)	σ_e, T_p, ψ_e
Gravitational / matter	$a \neq 0$	M, P, L, V, y, ψ_univ

The electron wavefunction ψ_e lives entirely in the thermal sector: it has $a = 0$ and scalars $= 1$. It is a dimensionless invariant assembled from temporal objects only—no mass enters. The universe wavefunction ψ_univ lives entirely in the gravitational sector: it requires M^4, which no thermal object can supply.

The sector boundary ($a = 0$ axis intersected with the $\theta = 0$ line at the origin $(0, 0)$) is the single point ψ_e itself. It is the unique object that is simultaneously mass-free, unit-number zero, and scalar-unity—the one object shared between both sectors.

7.4. Lattice Positions of the Three Levels

With the $\theta = 0$ lattice $\{(2n, n) : n \in \mathbb{Z}\}$ identified, the three oscillation levels map to specific lattice positions:

The relationship between the three levels is therefore exact in the lattice:

The level connection identity:

$$\psi_\mathrm{univ} \;=\; y^2 \cdot \psi_\mathrm{e} \quad \text{(in scalar content)}, \tag{38}$$

where $y = M^2 T$ is the Level 0 generator. The cosmological wavefunction is the electron invariant scaled by the square of the Planck simulation step. Electron and universe are separated by exactly two Planck steps in the $\theta = 0$ lattice.

This gives the lattice picture of the hierarchy:

Table 5: Lattice positions of the three oscillation levels on the $\theta = 0$ line $a = 2b$ in the M–T plane.

n	(a, b)	Object	Scalars	Role
0	(0, 0)	$\psi_{\mathrm{e}} = \sigma_e^3/(2T)$	1	Level 1 invariant. Static, mass-free, temporal. The identity element of the lattice.
1	(2, 1)	$y = M^2T$	r^{17}/v^8	Level 0 generator. The Planck step; one simulation increment. The lattice unit step vector.
2	(4, 2)	$\psi_{\mathrm{univ}} \sim \alpha^4 M^4 T^2$	r^{34}/v^{16}	Level 2 state density. Dynamic, mass-bearing, cosmological. Two generator steps from ψ_{e}.

$$\underbrace{(0,0)}_{\psi_{\mathrm{e}}} \xrightarrow{+y} \underbrace{(2,1)}_{y} \xrightarrow{+y} \underbrace{(4,2)}_{\psi_{\mathrm{univ}}} \xrightarrow{+y} \underbrace{(6,3)}_{?} \xrightarrow{+y} \cdots \tag{39}$$

7.5. Factorisation of the Degree-15 Extension

Examining the LCD values across all derived quantities reveals a factorisation:

- **LCD** $= 3$ (cube-root sector): σ_e, T_{p}. Both mass-free; both inhabit the thermal sector ($a = 0$). This is the *degree-3 temporal extension* of the integer M–T lattice.
- **LCD** $= 5$ (fifth-root sector): A (current), which has $a = -3/5$, $b = -2/5$, and $A^5 \propto M^{-3}T^{-2}$. This is the *degree-5 electromagnetic extension.*
- **LCD** $= 15 = \mathrm{lcm}(3, 5)$ (full sector): P, L, V—the quantities that carry both temporal structure (via T^b with denominator 15) and electromagnetic structure (via Ω).

The base-15 system factorises as a tower of extensions:

$$\mathbb{Z}[M, T] \subset \mathbb{Z}[M, T^{1/3}] \subset \mathbb{Z}[M^{1/5}, T^{1/15}] \xrightarrow{\Omega} \text{full SI}. \tag{40}$$

The first extension (degree 3) introduces cube roots of T, yielding σ_e and T_{p}. The second extension (degree 5 over degree 3, total degree 15) adds fifth roots and brings in Ω through P, completing the SI system. The integer $15 = 3 \times 5$ is not arbitrary: it is the product of the two irreducible extension degrees, each associated with a physically distinct sector.

7.6. The Open Lattice Point $n = 3$

The lattice chain in Eq. (39) does not terminate at $n = 2$. The next integer point is

$$n = 3: \quad (a, b) = (6, 3), \quad M^6T^3 = y^3, \quad \text{scalars} = r^{51}/v^{24}. \tag{41}$$

By the pattern established, $\psi_? \sim \alpha^k M^6T^3$ would represent a *third* oscillation level, two steps beyond the electron and one step beyond the universe. Its natural scale — M^6T^3 in natural units gives m_P^3 — would correspond to an energy cube, or equivalently to a trilinear gravitational coupling. Whether the framework supports a physical realisation at $n = 3$ remains an open question, but the lattice geometry requires its existence as a mathematically consistent site.

8. Summary: The Three Oscillation Levels

Table 6: Complete summary of the three oscillation levels.

Level	Scale	State function	Key structure
0	Planck step	$T = \pi$	Single geometric tick; base of the simulation loop.
1	Quantum scale	$\psi_e = \sigma_e^3/(2T)$	Electron as three magnetic monopoles $(AL)^3$ divided by time T. Scalars = 1; static invariant. Involves $a^3 \propto \alpha^{-3}$.
2	Cosmological	$\psi_{\text{univ}} \sim \alpha^4 M^4 T^2$	Universe as square of key operator $y^2 = (M^2T)^2$. Running state density; scalars = r^{34}/v^{16}. Closes at $t_{\text{age}} = t_{\text{end}} \propto \alpha^4$.

The three-level chain can be written as a single compact expression:

$$\underbrace{T = \pi}_{\text{Level 0}} \xrightarrow{\times (AL)^3} \underbrace{\psi_e \sim a^3 (AL)^3/T}_{\text{Level 1}} \xrightarrow{\times M^4T^2} \underbrace{\psi_{\text{univ}} \sim \alpha^4 M^4 T^2}_{\text{Level 2}} \tag{42}$$

and in terms of the generator and its powers:

$$\underbrace{r^{17}/v^8}_{n=1,\ \text{increment}} \longrightarrow \underbrace{\left(r^{17}/v^8\right)^2}_{n=2,\ \psi_{\text{univ}}\ \text{core}} \longrightarrow \underbrace{\alpha^4 \cdot \left(r^{17}/v^8\right)^2}_{\psi_{\text{univ}}} \tag{43}$$

9. Conclusion

We have established the following for Level 2 of the oscillation hierarchy:

1. **Central identity.** The ratio P^5/T_p^4 has unit number $\theta = 0$ and scalars r^{34}/v^{16}, identical to M^4T^2. This is the square of the key operator $y = M^2T$:
$$P^5/T_\mathrm{p}^4 = M^4T^2 = y^2.$$

2. **Universe wavefunction.** By analogy with ψ_e at Level 1, and incorporating the α^4 factor from the universe endpoint condition (4), the Level 2 state function is
$$\psi_\mathrm{univ} \ \sim\ \alpha^4 \cdot M^4T^2 \ \sim\ \alpha^4/G.$$
Unlike ψ_e, this is a dynamic (running) density rather than a static invariant: the oscillation completes only when $t_\mathrm{age} = t_\mathrm{end}$.

3. **The generator r^{17}/v^8.** This is the unique minimal $\theta = 0$ scalar combination of the two SI anchors r and v, formed by the cross-product of their unit numbers ($r^{17}/v^8 = r^{\theta(v)}/v^{\theta(r)}$). It is the scalar backbone of the key operator $y = \pi\, r^{17}/v^8$. All $\theta = 0$ scalars are integer powers of this generator. Its square, r^{34}/v^{16}, is the scalar content of ψ_univ. Numerically, its range across physical constants spans $\sim 10^{120}$, matching the temporal extent of the cosmological clock.

Together, the three oscillation levels constitute a complete base-15 hierarchy: from the single Planck tick at Level 0, through the fixed electromagnetic invariant at Level 1, to the evolving gravitational state density at Level 2. The generator r^{17}/v^8 is the thread that connects them all.

$$f(y) = \frac{v^8}{r^{17}} \ \sim\ 1.230016431 \times 10^{59}$$

We can then offer an observation on dimensional invariance: v and r are the numerical scalars tied to our human choice of SI units (meters, seconds, kilograms). If we were to meet aliens using a different system of units, their numerical values and units for v and r would not resemble ours. The numerical value $1.230016431 \times 10^{59}$ is an artifact of the SI system and this raises an interesting problem for the universe must be unit independent. This suggests that for any system of units there is an internal constraint whereby if we choose the value of v (for example as 300000 km/s or 186000 miles/s) then the value and units for r are fixed by f(y), which may now be defined as a universal constant built into the source code itself. Thus any system of units is permitted only 1 anchor (1 degree of freedom).

$$f(y) = \sim\ 1.230016426 \times 10^{59}$$

As we require MT+α to generate the physical constants, our f(y) may include an α^2 term (from T_p);

$$f(y)^2 = \frac{P^5}{T_P^4} = \frac{v^{16}}{\alpha^4 r^{34}}$$

Analysis of f(y) is continued in Part 4. of this series.

References

[1] *Planck Unit Scaffolding to Cosmic Microwave Background Correlation (a Simulation Hypothesis model)*, § Cosmological Constant. https://simulationuniverse.org/1-Planck-unit-CMB.html

[2] *Physical Constant Anomalies as Evidence of a Mathematical Universe*, § 5 The Base-15 Constraint; § 5.1 Unit Number Operations. https://simulationuniverse.org/6-Physical-constant-anomalies.html

[3] *Geometric Origin of Quarks*, https://simulationuniverse.org/7-Monopole-quarks.html