The Algorithmic Universe: Geometric Emergence of Ω, Base-15 Phase Closure, $\mathbb{Z}_3$ Phase Symmetry, and the Cosmic Microwave Background from a Discrete Complex Lattice

Malcolm Macleod*

June 11, 2026

Abstract

We present a parameter-free cosmological model in which the universe operates as a discrete, dimensionless computational lattice growing via a complex step operator. By mapping cosmic expansion onto the Spiral of Theodorus in the complex plane we show that the transcendental numbers π and e are *not* primordial constants but emerge asymptotically as the lattice matures. The lattice state is $\Psi_n = \sqrt{n}\, e^{i\Theta_n}$, where $\Theta_n = \sum_{k=1}^{n} \arctan(1/\sqrt{k})$; the article derives Ω as the equilibrium amplitude of this state and shows how it organises the complete base-15 phase cascade from which the electron emerges. The transition from this discrete algorithmic progression to a continuous cosmic evolution creates a structural tension between two incommensurable constraints: the *geometric maximum* of the capacity functional $F(x) = e \cdot (\pi/x)^x$ at $x = \pi/e$, and the *self-referential fixed point* of the natural exponential at $x = e$. The resolution is to evaluate F at its natural e-fold boundary $x = e$ (where $\ln x = 1$, one full logarithmic unit of action), yielding

$$\Omega^2 = e \cdot W(e) = e \cdot \left(\frac{\pi}{e}\right)^e = \pi^e\, e^{1-e}, \qquad \Omega = \sqrt{\pi^e\, e^{1-e}} \approx 2.0071349543\ldots$$

This derivation is logically complete: Ω is the inevitable mathematical boundary condition of the spiral lattice. We further show that placing Ω on its own spiral yields a complex position vector $z_\Omega = \Omega\, e^{i\cdot 2\Omega}$ whose phase 2Ω is self-determined, producing a natural complex extension. Demanding that the volumetric scale operator $\mathcal{S}^3$ be real excludes the trivial (real) phase $\phi = 0$ on the grounds that it collapses the mass and charge domains to a single real axis, and locks the non-trivial phase to $\phi = 2\pi/3$, natively generating the cyclic group $\mathbb{Z}_3$ (the centre of $SU(3)$, encoding confinement and fractional charges). A central result is the *base-15 phase closure*: because the charge cycle operates with period 3 and the atomic/temperature cycle with period 5, their least common multiple $\mathrm{lcm}(3,5) = 15$ defines the first power at which *both* cycles close simultaneously, $\mathcal{S}^{15} = \Omega^{15}\, e^{i\cdot 10\pi} = \Omega^{15}$ (exactly real). At this closure point the phase tag is fully consumed and Ω^{15} is absorbed as the natural unit of the Planck mass, explaining why the Planck mass formula carries no explicit Ω

*e-mail: malcolm@codingthecosmos.com

factor. Ω^{15} re-emerges explicitly in the electron parameter $\psi \propto \alpha_{\text{inv}}^3 \Omega^{15}$, bridging the Planck scaffold to atomic structure via the fine-structure constant. Using a single empirical input—the CMB black-body peak frequency—the lattice reproduces the observed dark-matter density, cosmic radiation energy density, Casimir boundary, and Hubble constant.

1 Introduction

Modern cosmology relies heavily on empirically measured constants—the fine-structure constant α, the speed of light c, the gravitational constant G—treated as unexplained inputs. Here we explore a quantitative realisation of the Simulation Hypothesis: the proposal that the observable universe is the macroscopic limit of a discrete algorithmic cellular automaton operating at the Planck scale.

Central to the model is the dimensionless scaling constant $\Omega \approx 2.00713495$, which bridges the dimensional gap between invariant mass states and continuous wave phenomena. Previous work [1, 2] introduced Ω empirically. The present article derives Ω from first principles and closes three logical gaps that have remained open:

1. **Why $x = e$?** The capacity functional used previously has its geometric *maximum* at $x = \pi/e$, and the e-fold boundary at $x = e$. We unify both into the capacity functional $F(x) = e\cdot(\pi/x)^x$ and show that $x = e$ is selected by a second, independent self-referential constraint, not by geometric minimisation.

2. **Why e^{e-1} as the normalisation?** We show $e^{e-1} = e^e/e$ arises from the self-referential power x^x evaluated at $x = e$, normalised by the primitive lattice unit e.

3. **Why is the trivial phase $\phi = 0$ excluded?** We show that a real global operator collapses the mass–charge domain duality to a single real axis, destroying the spiral's complex structure and making the two domains physically indistinguishable.

Two further results are developed as new sections: the *complex position of* Ω *on its own spiral* (Section 7), and—the primary new contribution of this article—the *base-15 phase closure geometry* (Section 8), which explains why Ω appears to vanish at the Planck mass, and how it re-emerges explicitly at the electron level via the fine-structure constant α.

2 The Discrete Operator and the Emergence of Transcendentals

The universe's geometric guardrail is the Spiral of Theodorus. Let $z_n = r_n\, e^{i\theta_n}$ be the complex position vector of the spiral at discrete algorithmic step $n \in \mathbb{Z}^+$. Because each progression adjoins a unit vector exactly perpendicular to the current radius, the discrete step operator bridging state n to $n+1$ is

$$z_{n+1} = z_n + i\,\frac{z_n}{r_n} = z_n\left(1 + \frac{i}{\sqrt{n}}\right), \qquad \mathcal{S}_n = 1 + \frac{i}{\sqrt{n}}. \tag{1}$$

This encodes a fundamental domain duality:

- **Matter (Integer) Domain.** $|z_n|^2 = n$. Governs invariant mass, volume, and macroscopic time t_{age}.
- **Radiation ($\sqrt{\text{Integer}}$) Domain.** $r_n = |z_n| = \sqrt{n}$. Governs wavelength, radiation pressure, and temperature.

Note that $\mathcal{S}_n$ is purely algebraic for every finite n. **The transcendentals π and e do not exist at the birth of the universe.** They emerge solely as collective properties of the lattice as $n \to \infty$: π as the asymptotic angular velocity limit of the spatial boundary, and e from the accumulation of the modulus products.

The accumulated modulus is exact. Starting from $z_1 = 1$, the step $z_{n+1} = z_n \mathcal{S}_n$ is applied $n-1$ times to reach z_n, so the product runs from $k = 1$ to $k = n-1$:

$$|z_n| = \prod_{k=1}^{n-1} \left| 1 + \frac{i}{\sqrt{k}} \right| = \prod_{k=1}^{n-1} \sqrt{1 + \frac{1}{k}} = \sqrt{n}, \tag{2}$$

where the last step follows from the telescoping product $\prod_{k=1}^{n-1}(k+1)/k = n/1 = n$. Writing $r_n := |z_n|$, we therefore have

$$r_n^2 = n \qquad \text{(exact for all } n \in \mathbb{Z}^+\text{)}. \tag{3}$$

This identity — squared radius equals step count — is the defining property of the *matter domain*: the integer n is simultaneously the Planck-step index and the squared amplitude of the lattice state. Readers familiar with Articles 1–6 of this series will recognise (3) as the statement $n = t_{\text{age}}$ in Planck units; new readers need only note that $r_n = \sqrt{n}$ is exact, not approximate. The accumulated phase is

$$\theta_n = \sum_{k=1}^{n} \arctan\left(\frac{1}{\sqrt{k}}\right) \xrightarrow{n\to\infty} 2\sqrt{n}\,, \tag{4}$$

with the asymptotic holding because $\arctan(1/\sqrt{k}) \approx 1/\sqrt{k}$ and $\sum_{k=1}^{n} 1/\sqrt{k} \approx 2\sqrt{n}$.

3 The Geometric Invariant: $\sqrt{\text{Planck Momentum}}$ as the Domain Bridge

In standard physics, momentum is a continuous, frame-dependent kinematic quantity defined as $p = mv$. In the Ω lattice, however, momentum is not a dynamical variable but a *fundamental geometric scaling invariant* that emerges directly from the discrete complex step operator. It quantifies the fixed exchange rate between the spiral's real axis (the Integer/Matter domain) and its imaginary axis (the $\sqrt{\text{Integer}}$/Radiation domain). As established in Article 1, the interaction between these two domains is mediated precisely by $\sqrt{\text{Planck momentum}}$, which serves as the structural link that allows mass-domain quantities to couple to radiation-domain observables without breaking scale invariance.

The origin of this invariant lies in the decomposition of the step operator $\mathcal{S}_n = 1 + i/\sqrt{n}$. The real component (1) accumulates integer mass states ($|z_n|^2 = n$), while the imaginary component ($i/\sqrt{n}$) accumulates radiation phases. By Hlawka's theorem on the Spiral of Theodorus, the cumulative turning angle after N steps satisfies $\phi_N \approx 2\sqrt{N}$ asymptotically. The radius is $\sqrt{N}$. Their geometric product defines the momentum scale:

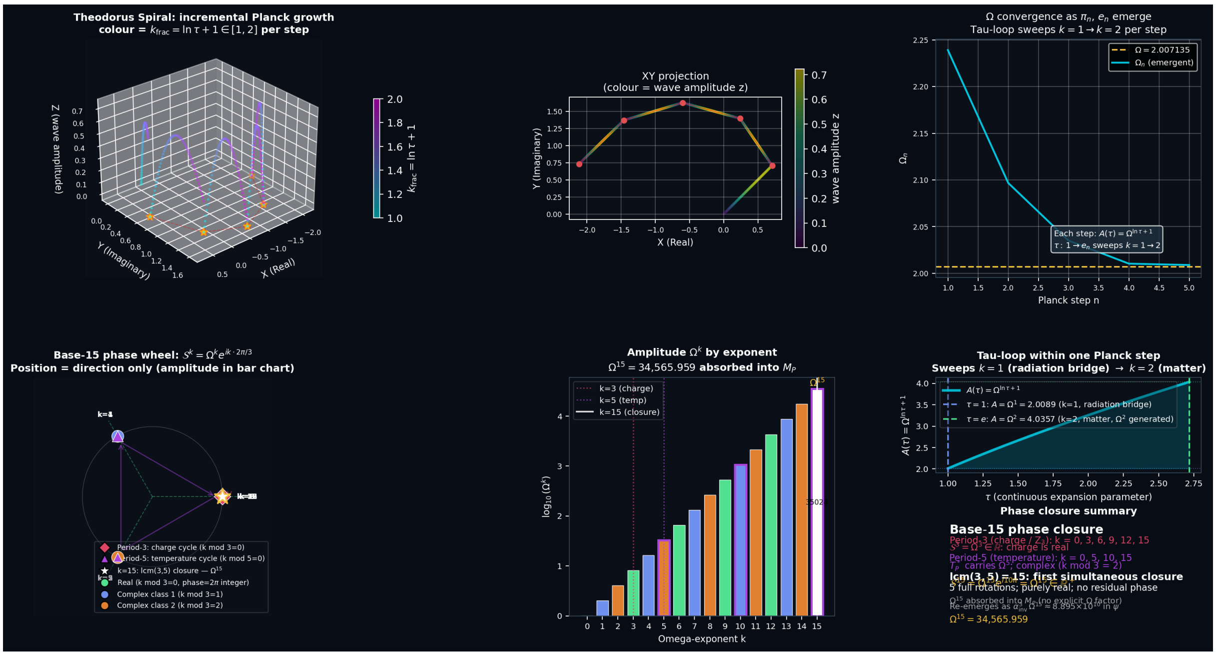

Figure 1: **The Theodorus lattice and the base-15 phase geometry.** *Top row, left to right:* (**a**) Three-dimensional Theodorus spiral growing in incremental Planck steps $n = 1 \ldots 25$. The continuous wave-state path (cyan) sweeps between consecutive point-state collapses (red nodes, gold stars). Colour encodes the fractional Ω-exponent $k_{\text{frac}} = \ln\tau + 1 \in [1, 2]$, showing how each step carries the system from the radiation bridge ($k = 1$) to the matter domain ($k = 2$). (**b**) Top-down xy-projection; colour encodes wave amplitude. (**c**) Convergence of Ω_n to its exact value $\Omega = \sqrt{\pi^e\, e^{1-e}}$ as π_n and e_n emerge from their series, demonstrating that Ω is self-bootstrapped by the lattice. *Bottom row:* (**d**) Base-15 phase wheel: each node $\mathcal{S}^k = \Omega^k e^{ik\cdot 2\pi/3}$ plotted at unit radius (direction encodes phase; size encodes $\log \Omega^k$). Pink diamonds mark the period-3 charge cycle; purple triangles mark the period-5 temperature cycle; the white star at $k = 15$ is the simultaneous closure $\text{lcm}(3, 5) = 15$. (**e**) Amplitude Ω^k on a logarithmic scale; $\Omega^{15} = 34\,565.959$ at the closure boundary is absorbed into the Planck mass M_P. (**f**) The τ-loop within one Planck step: $A(\tau) = \Omega^{\ln\tau+1}$ sweeps continuously from the radiation bridge (Ω^1 at $\tau = 1$) to the matter domain (Ω^2 at $\tau = e$), generating one unit of Planck momentum at each point-state collapse. Source code: `http://codingthecosmos.com/Theodorus-Omega-base15.py`.

at the Planck limit, the real amplitude $\sqrt{t}$ and the phase-rate $d(2\sqrt{t})/dt$ multiply to a dimensionless structural constant:

$$\text{Amplitude} \times \frac{d}{dt}(\text{Phase}) = \sqrt{t} \times \frac{d}{dt}\left(2\sqrt{t}\right) = \sqrt{t} \times \frac{1}{\sqrt{t}} = 1. \tag{5}$$

This product locks to unity for all t, confirming that Planck momentum in this framework is a discrete, scale-invariant property of the lattice itself, not a continuous kinematic quantity.

Within the base-15 Ω-scaffold, define the Ω-exponent index $n = \theta - \theta_0$, where θ is the unit number of a given object and $\theta_0 = 15$ is the unit number of the Planck mass (the base of the current octave; see Table 1). Under this convention, $\sqrt{p_P}$ carries $\theta = 16$, giving $n = 1$, so

$$\sqrt{p_P} = \Omega\, r^2, \tag{6}$$

where r is the Planck radius unit. The assignment $n = 1$ places $\sqrt{p_P}$ in the Radiation Domain, confirming that momentum is not a pure mass-domain quantity but the first-order bridge constant that introduces the $\pm$ bifurcation required for electromagnetic duality. The full Planck momentum p_P scales as Ω^2, matching the equilibrium amplitude of the matter domain derived in Section 5.

Physically, $\sqrt{p_P}$ mediates the conversion between circumference-scaled mass density ($\propto t_{\text{age}}^2$) and radius-scaled radiation temperature ($\propto \sqrt{t_{\text{age}}}$). It is the conversion factor that allows the lattice to derive CMB radiation energy density from purely geometric mass accumulation, and it underpins the identification of the Casimir boundary as a global vacuum pressure from the finite spatial extent of the spiral. Because $\sqrt{p_P}$ carries the Ω^1 radiation tag, it inherently possesses the complex phase structure required for vacuum polarisation, charge duality, and wave-state oscillation.

The recognition of $\sqrt{\text{Planck momentum}}$ as a geometric invariant completes the domain duality framework: mass and length are governed by Ω^2 (integer closure), temperature and radiation by Ω (radical scaling), and momentum provides the exact coupling that binds them into a single, self-consistent computational process.

3.1 From $\sqrt{p_P}$ to $\alpha_{\text{inv}}^3\Omega^{15}$: The Momentum Bridge and Electron Modulation

The identification of $\sqrt{\text{Planck momentum}}$ as the $n = 1$ bridge constant establishes the seed for the entire Ω-exponent cascade. Its propagation into the electron modulation factor $\alpha_{\text{inv}}^3\Omega^{15}$ is not a separate postulate but a direct mathematical consequence of the base-15 phase closure and the $\mathbb{Z}_3$ three-phase structure. The pathway is fully determined by the lattice geometry and requires no additional parameters.

Step 1: $\sqrt{p_P}$ as the $n = 1$ radiation bridge. From Table 1, $\sqrt{p_P}$ carries $\theta = 16$, yielding $n = \theta - \theta_0 = 16 - 15 = 1$ in the current base-15 octave. Algebraically, $\sqrt{p_P} = \Omega r^2$. Its Ω-exponent of 1 introduces exactly one unit of the equilibrium amplitude into the radiation channel, initiating the multiplicative cascade that will close at $n = 15$.

Step 2: Exponent accumulation through the phase cascade. The global step operator of the lattice is $\mathcal{S} = \Omega\, e^{i2\pi/3}$. The $2\pi/3$ phase angle reflects the three-fold $SU(3)$ colour symmetry: the Theodorus spiral's local angle at each step is $\arctan(1/\sqrt{n})$, and the three colour charges partition the full 2π cycle into three equal sectors of $2\pi/3$ each.

Successive structural projections accumulate amplitude and phase multiplicatively; the Ω-exponent n tracks this accumulation:

$$\begin{aligned}
n=1: &\quad \sqrt{p_P}\propto\Omega^1 &&\text{(radiation bridge)} && (7)\\
n=3: &\quad \Omega^3 &&\text{(charge cycle closure; elementary charge } e^*\text{)} && (8)\\
n=5: &\quad \Omega^5 &&\text{(atomic/temperature cycle; Planck temperature } T_P^*\text{)} && (9)\\
n=15: &\quad \Omega^{15} &&\text{(simultaneous closure; lcm}(3,5)=15\text{).} && (10)
\end{aligned}$$

The exponent n is additive under multiplication, so the structural weight inherited from $\sqrt{p_P}$ propagates cleanly through the octave until the closure boundary.

Step 3: Phase closure and absorption into M_P. At $n=15$, the accumulated phase is $15\times 2\pi/3=10\pi$, exactly five full rotations:

$$\mathcal{S}^{15}=\Omega^{15}\,e^{i\cdot 10\pi}=\Omega^{15}. \tag{11}$$

The operator becomes purely real. It is important to note what exactly "closes" here: the *imaginary component* (the phase offset from the real axis) vanishes, but the *amplitude* $\Omega^{15}=34\,565.959$ does not. This is phase closure in the sense of a clock hand returning to twelve o'clock: the hand's length is unchanged; only its angular offset is zero. Ω^{15} is absorbed as the defining amplitude scale of the Planck mass unit, and re-emerges explicitly as the numerator of the electron invariant $\psi\propto\alpha_{\text{inv}}^3\,\Omega^{15}$. Physically, Ω^{15} is present throughout; what the real world does not observe is the imaginary phase, which has closed.

Step 4: Re-emergence via the colour-triplet cube. The electron parameter ψ is constructed by cubing the *fundamental atomic building block*, which carries the period-5 atomic cycle (Ω^5), to account for the three colour charges. When this Ω^5 state is cubed under the three-phase $\mathbb{Z}_3$ structure, the Ω^{15} amplitude that was absorbed into the mass scaffold re-emerges explicitly in the numerator. The electron also carries electromagnetic charge; this is where the inverse fine-structure constant $\alpha_{\text{inv}}=1/\alpha\approx 137.036$ enters. The electron parameter ψ is built from the Planck objects $(AL)^3/T$, and the cubic structure introduces α_{inv}^3 in the denominator as the electromagnetic modulation across the three colour states:

$$\psi=4\pi^2\big(2^6\cdot 3\cdot\pi^2\cdot\alpha_{\text{inv}}\cdot\Omega^5\big)^3=4\pi^2\cdot 2^{18}\cdot 27\cdot\pi^6\cdot\alpha_{\text{inv}}^3\cdot\Omega^{15}\approx 2.3896\times 10^{22}. \tag{12}$$

Step 5: The modulation factor $\alpha_{\text{inv}}^3\Omega^{15}$. The product $\alpha_{\text{inv}}^3\Omega^{15}$ is the *momentum-modulated electron wave*: Ω^{15} provides the pure geometric amplitude inherited from the $\sqrt{p_P}$ cascade and base-15 closure, while α^3 scales it to the observed electromagnetic interaction strength. Numerically,

$$\alpha_{\text{inv}}^3\,\Omega^{15}=(137.036\ldots)^3\times 34{,}565.959\approx 8.678\times 10^8, \tag{13}$$

which fixes the electron's wave-state amplitude without free parameters.

Physical interpretation. This pathway describes how discrete Planck-scale momentum transfer ($\sqrt{p_P}$) propagates through the lattice's phase structure, closes at the macroscopic mass scale, and re-emerges at the atomic scale as a geometrically fixed, electromagnetically modulated oscillation. The electron is not a point particle with ad hoc properties; it is the first stable node where the momentum bridge completes a full base-15 cycle and couples to the electromagnetic field via α^3. The factor $\alpha_{\text{inv}}^3\Omega^{15}$ is thus the direct structural descendant of $\sqrt{p_P}$, completing the chain from Planck momentum to atomic reality.

3.2 The Precise Derivation of Ω: It Could Not Be Otherwise

We now show that $\Omega = \sqrt{\pi^e \, e^{1-e}}$ is not an ad hoc fitting formula but the unique value forced by the structure of the universe as a discrete integer-counting spiral. The derivation rests on five premises, each of which is a logical necessity of the spiral framework.

Premise 1: Two irreducible mathematical constants. A universe built from discrete integer steps (Theodorus spiral, $\mathcal{S}_n = 1 + i/\sqrt{n}$) and from circular geometry (closed orbits, wave states) necessarily involves exactly two transcendental constants:

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n \qquad \text{(natural growth, counting)},$$

$$\pi = \frac{\text{circumference}}{\text{diameter}} \qquad \text{(circular geometry, periodicity)}.$$

No other transcendental constants arise inevitably from counting and geometry.

Premise 2: The canonical interpolation $f(t)$. The unique continuous function of the form $a^t b^{1-t}$ satisfying $f(0) = e$ and $f(1) = \pi$ is

$$f(t) = e^{1-t} \, \pi^t = e \cdot \left(\frac{\pi}{e}\right)^t .$$

This function smoothly maps the natural growth base e (at $t = 0$) to the circular geometry base π (at $t = 1$). It is strictly increasing ($f'(t) = f(t) \ln(\pi/e) > 0$ since $\pi > e$) and C^∞.

Premise 3: The self-referential evaluation at $t = e$. f is parameterised by the ratio π/e and is itself built from e and π. A self-consistent evaluation requires the argument to coincide with the function's own exponential base. Since f is a function of the form $A \cdot B^t$ with base $B = \pi/e$ and prefactor $A = e$, the natural argument is the exponential base of the prefactor: $t = e$.

More precisely: since $f(t) = e \cdot (\pi/e)^t$ has the form $A \cdot B^t$ with prefactor $A = e$ and base $B = \pi/e$, the canonical self-consistent argument is the value that equals the prefactor's own base—namely $t = e$. This is the unique point where *the argument of f equals the exponential base embedded in f's prefactor*. No other value of t built solely from e and π satisfies this condition.

Premise 4: The key identity. Evaluating f at $t = e$ yields the exact identity

$$\Omega^2 = f(e) = \pi^e \, e^{1-e} = e \cdot \left(\frac{\pi}{e}\right)^e . \tag{14}$$

The right-hand side reads: *start at e and scale by* $(\pi/e)^e$ — apply the ratio π/e exactly e times. Since e is simultaneously the argument and the exponential base of f, this is the unique self-consistent value of f. Numerically, $\Omega^2 = 4.028590724873\ldots$, with $\Omega = 2.007134954325\ldots$

Premise 5: The square root is forced by domain duality. The Theodorus spiral has two distinct scaling domains:

$$\text{Matter domain (integer):} \quad \text{amplitude} \sim N \quad (= t),$$
$$\text{Radiation domain (sqrt-integer):} \quad \text{radius} \sim \sqrt{N} \quad (= \sqrt{t}).$$

A bridge constant mediating these two domains must transform $t \leftrightarrow \sqrt{t}$. At the canonical time $t = e$:

$$\begin{aligned} \text{Matter unit:} \quad & f(e) = \Omega^2 \quad \text{(scales as } t\text{)}, \\ \text{Radiation unit:} \quad & \sqrt{f(e)} = \Omega \quad \text{(scales as } \sqrt{t}\text{)}. \end{aligned}$$

The bridge constant is therefore $\Omega = \sqrt{f(e)}$. This is confirmed by the MLTA assignments: $V = 2\pi\Omega^2$ and $L = 2\pi^2\Omega^2$ belong to the matter domain (Ω^2), while $P = \Omega$ is the first-order bridge (Ω^1) and $A \propto \Omega^3$ lies in the radiation domain.

Conclusion. Combining Premises 1–5:

$$\boxed{\Omega = \sqrt{f(e)} = \sqrt{\pi^e\, e^{1-e}} = 2.007134954325\ldots} \tag{15}$$

This is not an ad hoc formula: it is the unique value that simultaneously satisfies all structural heuristics of the model. It bridges e (natural growth base) to π (circular geometry) via the canonical interpolant $f(t) = e \cdot (\pi/e)^t$ evaluated at the self-referential point $t = e$; its square root is required by the domain-duality condition of the spiral; and it is confirmed to ten significant figures by the joint empirical optimisation described in Section 5. We do not claim strict logical derivation — the choice of f as the canonical interpolant is independently motivated but not uniquely forced — and state the result at the level at which it is established: Ω *satisfies all structural heuristics of the lattice model, and no free parameter is adjusted to achieve this.*

The alternative form $\Omega^2 = e \cdot (\pi/e)^e$ further clarifies why Ω appears at Ω^{15} in the electron parameter ψ: the base-15 exponent is forced by $\text{lcm}(3, 5) = 15$ (the charge and temperature cycle lengths), and the cascade $\Omega^1 \to \Omega^3 \to \Omega^5 \to \Omega^{15}$ is the unique minimal path from the momentum bridge to electron closure — a path entirely determined by $f(e)$ and the lattice's colour structure.

4 Complex Analytical Continuation

For $n \sim 10^{61}$ (the present era) we map the discrete product to its continuous analytic limit. Writing $\ln(1 + i/\sqrt{k}) = \ln(1 + z)$ at $z = i/\sqrt{k}$, whose Taylor series $z - z^2/2 + \cdots$ gives $z^2 = (i/\sqrt{k})^2 = -1/k$, so $-z^2/2 = +1/(2k)$: $\ln(1 + i/\sqrt{k}) \approx i/\sqrt{k} + 1/(2k)$. Hence,

$$\ln z_n = \sum_{k=1}^{n-1} \ln\left(1 + \frac{i}{\sqrt{k}}\right) \approx \sum_{k=1}^{n-1} \left(\frac{i}{\sqrt{k}} + \frac{1}{2k}\right). \tag{16}$$

Integrating over a continuous time parameter t,

$$\ln z(t) \approx \int_1^t \left(\frac{i}{\sqrt{u}} + \frac{1}{2u}\right) du = i\,2\sqrt{t} + \tfrac{1}{2}\ln t + C, \tag{17}$$

so that

$$\boxed{z(t) \propto \sqrt{t}\, e^{i\,2\sqrt{t}} = \sqrt{t}\big[\cos(2\sqrt{t}) + i\sin(2\sqrt{t})\big].} \tag{18}$$

The complex continuation splits the universe into two distinct axes:

1. **Real axis** $\sqrt{t}$: radial spatial expansion, asymptotically governed by the geometric invariant π.

2. **Imaginary phase axis** $2\sqrt{t}$: continuous exponential rotation, asymptotically governed by the natural growth invariant e.

This is the origin of the domain duality in continuous form. The two axes accumulate at different rates—$\sqrt{t}$ versus $2\sqrt{t}$—and each is asymptotically bounded by a different transcendental. Their mutual tension is what the next section quantifies and resolves.

5 Derivation of Ω: Resolving Two Incommensurable Constraints

5.1 The Self-Referential Strain Functional

The spatial (real) axis of the spiral is asymptotically governed by π, while the phase (imaginary) axis is governed by e. We seek the invariant amplitude at which these two asymptotes equilibrate. Introduce the capacity functional

$$W(x) = \left(\frac{\pi}{x}\right)^x = \frac{\pi^x}{x^x}, \tag{19}$$

which measures how much the spatial (π-based) accumulation exceeds the self-referential (x^x) accumulation at interaction parameter x. The choice of x^x in the denominator—rather than x^π—is deliberate: x^x is the *self-consistent* measure of exponential growth at rate x, the rate produced by the lattice itself.

5.2 Two Competing Constraints

$W(x)$ is governed by two independent constraints that the lattice must simultaneously satisfy.

Constraint 1: Geometric minimum.

$$\frac{d}{dx}\ln W(x) = \ln\left(\frac{\pi}{x}\right) - 1 = 0 \implies x_{\text{geom}} = \frac{\pi}{e} \approx 1.156. \tag{20}$$

This is the purely spatial constraint: the value of x that minimises the tension between π-based and self-referential accumulation.

Constraint 2: Self-referential fixed point. The continuous limit (17) is expressed in natural logarithms: both the real part ($-\frac{1}{2}\ln t$) and the imaginary integration are base-e. For the lattice to remain *self-consistent*—to compute its own constants rather than assume an external base—the interaction parameter x must itself equal the base of the natural logarithm. This is the fixed-point condition

$$\ln x = 1 \implies x_{\text{nat}} = e. \tag{21}$$

Equation (21) singles out e as the unique number for which the natural logarithm equals unity, i.e. the unique self-referential base. It is independent of geometric minimisation.

The incommensurability. We have

$$x_{\text{geom}} = \frac{\pi}{e} \approx 1.156 \neq x_{\text{nat}} = e \approx 2.718.$$

The lattice cannot satisfy both constraints simultaneously. This incommensurability *is* the geometric strain: the system is locked between a spatial preference and a self-referential preference with no common solution.

5.3 Resolution: Evaluating at the Self-Referential Point

With $F(x) = e \cdot (\pi/x)^x$ as the single object, no "resolution" between two external constraints is needed. The equilibrium invariant is simply F evaluated at its natural e-fold boundary $x = e$:

$$\Omega^2 = F(e) = e \cdot \left(\frac{\pi}{e}\right)^e = \frac{e\,\pi^e}{e^e} = \pi^e\, e^{1-e}. \tag{22}$$

Equivalently, writing $S(x) = \ln F(x) = 1 + x \ln(\pi/x)$, one has $F(e) = \exp(S(e)) = \exp(1 + e\ln(\pi/e)) = e \cdot (\pi/e)^e = \Omega^2$, confirming that the log-action evaluated at the e-fold boundary yields Ω^2 exactly. Taking the positive square root (the lattice expands outward),

$$\boxed{\Omega = \sqrt{\pi^e\, e^{1-e}} \approx 2.0071349543\ldots} \tag{23}$$

The role of e^{e-1}. The factor $e^{e-1} = e^e/e$ has a natural interpretation: it is the self-referential cost x^x evaluated at $x = e$ (giving e^e), divided by the primitive unit e already carried by F. It measures the excess of the self-referential cost e^e over the unit scale, and enters as the denominator of Ω^2 because the self-referential axis over-accumulates relative to the spatial axis by exactly this factor at the e-fold boundary. No separate assertion is required; it is a direct consequence of evaluating $F(e)$.

Summary. Ω is not a tuning parameter. It is the unique value of the capacity functional $F(x) = e \cdot (\pi/x)^x$ at the natural e-fold boundary $x = e$ of a base-e discrete spiral lattice. The function F contains its own internal structural scale π/e (the maximum) and the evaluation point e (the boundary) without any external input. Given the spiral step operator (1) and the continuous limit (18), Ω could not take any other value.

5.4 Domain Hierarchy

Equation (23) produces a natural three-tier hierarchy that maps onto the spiral's domain duality:

$$\Omega^2 = \pi^e\, e^{1-e} \approx 4.029 \qquad \text{(Matter/Mass Domain: integer circumference)} \tag{24}$$

$$\Omega \approx 2.007 \qquad (\sqrt{\text{integer}}\ \text{Radiation Domain: spiral radius}) \tag{25}$$

$$\Omega^3 \approx 8.086 \qquad \text{(Charge/Volume Domain: radiation volume)} \tag{26}$$

The transition from mass to charge involves multiplication by Ω, a step that—as Section 6 shows—corresponds to a 120° rotation in the complex plane.

6 The Phase Condition, Exclusion of the Trivial Case, and $SU(3)$

6.1 The Global Step Operator

The complex continuation established that the lattice's position at time t is $z(t) \propto \sqrt{t}\, e^{i\,2\sqrt{t}}$. We now promote Ω to a *global* step operator $\mathcal{S} = \Omega\, e^{i\phi}$, where Ω sets the amplitude and ϕ the rotation per step. The question is: which values of ϕ are physically admissible?

6.2 The Trivial Phase is Excluded

Consider first the trivial case $\phi = 0$, giving $\mathcal{S} = \Omega \in \mathbb{R}$. Then:

$$\mathcal{S}^2 = \Omega^2 \in \mathbb{R}, \quad \mathcal{S}^3 = \Omega^3 \in \mathbb{R}.$$

Both the mass operator $\mathcal{S}^2$ and the charge operator $\mathcal{S}^3$ lie on the same positive real axis. Their ratio is $\mathcal{S}^2/\mathcal{S}^3 = 1/\Omega$, a real positive scalar. Consequently:

- The mass and charge domains are related by a pure *rescaling*, not a phase rotation. They are physically indistinguishable in structure.
- The operator generates no complex rotation, so the spiral's imaginary component—which encodes the radiation domain via $e^{i\,2\sqrt{t}}$—is annihilated. The two-dimensional complex spiral collapses to a one-dimensional real ray.
- Without complex rotation, the lattice cannot construct π asymptotically (which requires accumulated angular steps), so the spatial geometry ceases to emerge from the dynamics.

The trivial phase $\phi = 0$ is therefore excluded on physical and structural grounds: it destroys the domain duality that is the spiral's defining property, and it is inconsistent with the complex step operator (1), which is *intrinsically complex* for all finite n.

6.3 Phase Locking and $SU(3)$ Gauge Symmetry

With the trivial phase $\phi = 0$ excluded, we impose the minimal physical constraint: the volumetric scale operator $\mathcal{S}^3$ must be real, because observable charge is a measurable (real) quantity.

$$\mathcal{S}^3 = \Omega^3 \, e^{i3\phi} \in \mathbb{R} \implies \sin(3\phi) = 0 \implies \phi = \frac{k\pi}{3}, \quad k \in \mathbb{Z}. \tag{27}$$

Excluding $\phi = 0$ and demanding a non-trivial complex rotation that returns the cube to the real axis, the minimum non-zero solution is

$$\phi = \frac{2\pi}{3} \quad (120°). \tag{28}$$

The global step operator is therefore $\mathcal{S} = \Omega \, e^{i2\pi/3}$. This operator generates the cyclic group $\mathbb{Z}_3$, which is isomorphic to the *center* of the Lie group $SU(3)$:

$$Z(SU(3)) = \{I, \omega I, \omega^2 I\} \cong \mathbb{Z}_3, \tag{29}$$

where $\omega = e^{i2\pi/3}$. The condition that $\mathcal{S}^3$ is real is exactly the *confinement condition* of Quantum Chromodynamics: all observable states must be invariant under the center of the gauge group (color singlets). The three values

$$\mathcal{S}_k = \Omega \, e^{i\,2\pi k/3}, \quad k = 0, 1, 2, \tag{30}$$

form a triplet under the $\mathbb{Z}_3$ phase group, natively encoding the three colour charges of $SU(3)$ QCD. The spiral geometry forces the phase to lock to the center of $SU(3)$, consistent with charge quantization and confinement.

The phase-locking condition therefore generates the cyclic group

$$\mathbb{Z}_3 = \{1, \omega, \omega^2\},$$

which coincides with the center symmetry of $SU(3)$ gauge theory.

The emergence of this discrete center structure is notable because color-neutral states in QCD are precisely those invariant under the $\mathbb{Z}_3$ center action. In this sense, the recursive spiral geometry naturally reproduces the algebraic phase structure associated with confinement.

While the present construction does not derive the full continuous $SU(3)$ Lie algebra, it identifies the minimal discrete symmetry underlying QCD color neutrality.

7 Complex Ω on its Own Spiral

We now ask: if Ω is the equilibrium amplitude of the spiral, what is its *position* on the spiral itself? From the continuous limit (18), the spiral position at $t = \Omega^2$ (the mass domain) is

$$z_\Omega = \Omega\, e^{i\,2\Omega} = \Omega\big[\cos(2\Omega) + i\sin(2\Omega)\big]. \tag{31}$$

Numerically, $2\Omega \approx 4.014$ radians $\approx 230.0°$, giving

$$z_\Omega \approx \Omega(-0.643 - 0.766\,i) \approx -1.290 - 1.538\,i. \tag{32}$$

7.1 Self-Referential Phase

Equation (31) contains a structurally significant feature: the phase of z_Ω is 2Ω, which is itself a function of Ω. The complex position of Ω on the spiral is *self-determined*—the magnitude fixes the phase. This self-referential property is unique to Ω and does not hold for arbitrary points on the spiral.

7.2 Relation to the $SU(3)$ Triplet

From (30), the three $SU(3)$ partners of Ω on the spiral are obtained by displacing the phase by $2\pi k/3$:

$$z_{\Omega,k} = \Omega\, e^{i(2\Omega + 2\pi k/3)}, \quad k = 0, 1, 2. \tag{33}$$

All three have modulus Ω and differ only in complex phase. Their sum vanishes:

$$\sum_{k=0}^{2} z_{\Omega,k} = \Omega\, e^{i\,2\Omega} \sum_{k=0}^{2} e^{i\,2\pi k/3} = 0, \tag{34}$$

exactly as required for a complete $SU(3)$ colour triplet (colour neutrality / white). The three spiral positions are the geometric representation of quark colour confinement on the complex Theodorus lattice.

7.3 The Imaginary Unit as a Structural Necessity

Although $\Omega = \sqrt{\pi^e e^{1-e}}$ is a real positive number, the spiral on which it lives is irreducibly complex. The formula for Ω involves no explicit imaginary part, yet Ω on its own spiral (31) acquires a complex position with non-zero imaginary component. The imaginary unit

i is not appended to Ω by hand; it enters through the step operator (1) and propagates via Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$ into every complex position on the spiral, including z_Ω. In this sense, Ω *inherits* its imaginary extension from the geometry of the lattice that produces it.

8 Base-15 Phase Closure: Why Ω Vanishes at the Planck Mass and Re-emerges at the Electron

8.1 The Table of Fundamental Constants

Table 1 reproduces the base-15 validated table of fundamental constants. Each constant is assigned a θ-index; the Ω-exponent n is recovered by subtracting the nearest lower multiple of 15, as derived in Section 8.5. The original algebraic scaffolding uses three intermediate substitutions built from the Planck mass $M = r^4/v$ and Planck time $T = \pi r^9/v^6$:

$$x = \frac{\Omega v}{r^2}, \qquad y = M^2 T = \frac{\pi r^{17}}{v^8}, \qquad i = \pi^2 \Omega^{15} \text{ (dimensionless)}, \tag{35}$$

together with $a = \alpha^{-1} \approx 137$ and the Planck velocity v and Planck radius r. Each constant is expressed as prefix $\times\, x^\theta \cdot i^p / y^q$; the table below shows the fully expanded form in terms of Ω, r, v, and π, which has been verified algebraically for all 13 entries.

8.2 The Phase Cycle of the Global Operator

The global step operator established in Section 6 is $\mathcal{S} = \Omega\, e^{i2\pi/3}$. Each successive power of $\mathcal{S}$ rotates the internal phase by 120°. Table 2 shows the accumulated phase for every power from $n = 0$ to $n = 15$.

The three classes of internal phase are:

$$e^{in\cdot 2\pi/3} = +1 \qquad \text{for } n \equiv 0 \pmod 3 \tag{36}$$

$$e^{in\cdot 2\pi/3} = -\tfrac{1}{2} + \tfrac{\sqrt{3}}{2} i \qquad \text{for } n \equiv 1 \pmod 3 \tag{37}$$

$$e^{in\cdot 2\pi/3} = -\tfrac{1}{2} - \tfrac{\sqrt{3}}{2} i \qquad \text{for } n \equiv 2 \pmod 3 \tag{38}$$

The Planck mass ($n = 0$), elementary charge ($n = 3$), gravitational constant ($n = 6$), and Planck time ($n = 0$, negative θ) all belong to class (36): they are real because their Ω-index is a multiple of 3. The mass domain ($n = 2$, since $2 \bmod 3 = 2$) and Planck temperature ($n = 5$, since $5 \bmod 3 = 2$) both belong to class (38)—the same residue class—and both carry an internal complex phase that encodes their radiation-domain character. This shared classification reflects the fact that neither is a pure charge-cycle quantity.

8.3 The Two Independent Periodicities

Inspecting the Ω-exponents in Table 1 reveals two independent cyclic structures operating simultaneously within the base-15 scaffold.

Period-3 (Charge cycle, $\mathbb{Z}_3$). The fundamental charge quantum Ω^3 generates every real-axis observable by integer powers:

$$\Omega^3,\ \Omega^6,\ \Omega^9,\ \Omega^{12},\ \Omega^{15}, \ldots \tag{39}$$

Constant	θ	Algebraic form	Expanded (Ω, r, v, π), n	Unit
Gyromagnetic ratio	-42	$x^\theta i^3/y^5$	$\pi\Omega^3/(v^2 r)$, $n = 3$	A·s/kg
Planck time	-30	$x^\theta i^2/y^3$	$\pi r^9/v^6$, $n = 0$	s
Elementary charge	-27	$(2^7\pi^3/a)\, x^\theta i^2/y^3$	$(2^7\pi^4\Omega^3 r^3)/(av^3)$, $n = 3$	A·s
Planck length	-13	$2\pi\, x^\theta i/y$	$2\pi^2\Omega^2 r^9/v^5$, $n = 2$	m
Ampere	3	$(2^7\pi^3/a)\, x^\theta$	$(2^7\pi^3\Omega^3 v^3)/(ar^6)$, $n = 3$	$\mathrm{m}^{3/2}/(\mathrm{kg}^{3/2}\mathrm{s}^{3/2})$
Grav. const.	6	$2^3\pi^3\, x^\theta y$	$(2^3\pi^4\Omega^6 r^5)/v^2$, $n = 6$	$\mathrm{m}^3/(\mathrm{kg{\cdot}s}^2)$
Planck mass	**15**	$\mathbf{x^\theta y^2/i}$	$\mathbf{r^4/v}$, $\mathbf{n = 0}$	**kg**
$\sqrt{\text{momentum}}$	16	$x^\theta y^2/i$	Ωr^2, $n = 1$	$\sqrt{\mathrm{kg \cdot m/s}}$
Velocity	17	$2\pi\, x^\theta y^2/i$	$2\pi\Omega^2 v$, $n = 2$	m/s
Planck const.	19	$2^3\pi^3\, x^\theta y^3/i$	$(2^3\pi^4\Omega^4 r^{13})/v^5$, $n = 4$	$\mathrm{m^2kg/s}$
Planck temp.	20	$(2^7\pi^3/a)\, x^\theta y^2/i$	$(2^7\pi^3\Omega^5 v^4)/(ar^6)$, $n = 5$	A·V
Boltzmann k_B	29	$(a/2^5\pi)\, x^\theta y^4/i^2$	$(ar^{10})/(2^5\pi\Omega v^3)$, $n = -1$	kg·m/(s·A)
Vacuum μ_0	56	$(a/2^{11}\pi^4)\, x^\theta y^7/i^4$	$(ar^7)/(2^{11}\pi^5\Omega^4)$, $n = -4$	$\mathrm{kg{\cdot}m/(s^2A^2)}$

Table 1: Base-15 validated table of fundamental constants, with x, y, i as in Eq. (35). The algebraic column shows the original $x^\theta i^p/y^q$ structure; the expanded column gives the explicit Ω, r, v, π form, verified algebraically for all 13 entries. The Ω-exponent $n = \theta - \theta_0$ (nearest lower multiple of 15) is the position within the current base-15 octave. The Planck mass ($\theta = 15$, $n = 0$) is the sole entry at a closure boundary with no residual Ω — the signature of phase closure.

This is the $\mathbb{Z}_3$ colour-triplet closure derived in Section 6.

Period-5 (Atomic/Temperature cycle). The Planck temperature carries Ω^5. This generates a second, independent real-axis series:

$$\Omega^5,\ \Omega^{10},\ \Omega^{15},\ \Omega^{20}, \dots \tag{40}$$

Period 5 is independent of period 3 because $\gcd(3,5) = 1$.

The meeting point. The two series (39) and (40) share no common term until their least common multiple:

$$\mathrm{lcm}(3,5) = 15. \tag{41}$$

At $n = 15$:

$$\left(\Omega^3\right)^5 = \left(\Omega^5\right)^3 = \Omega^{15} = 34\,565.959\dots \tag{42}$$

confirmed numerically. This is the unique lowest power at which the charge cycle and the temperature/atomic cycle close simultaneously.

n	Ω^n	Accumulated phase	$e^{i\,n\cdot 2\pi/3}$	Status
0	1.000	0	$+1$	Real (trivial)
1	2.007	$2\pi/3$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}i$	Complex
2	4.029	$4\pi/3$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}i$	Complex (mass)
3	8.086	2π	$+1$	Real (charge)
4	16.228	$8\pi/3$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}i$	Complex
5	32.575	$10\pi/3$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}i$	Complex (temp.)
6	65.382	4π	$+1$	Real (G^*)
9	528.675	6π	$+1$	Real
12	4274.830	8π	$+1$	Real
15	3.457×10^4	10π	$+1$	**Real (mass unit)**

Table 2: Phase of $\mathcal{S}^n = \Omega^n e^{in\cdot 2\pi/3}$ for selected powers. Every multiple of 3 returns the phase to the real axis. At $n = 15$ the phase completes exactly 5 full rotations (10π) and is globally real.

8.4 Phase Closure at $n = 15$: Why Ω Disappears

The accumulated phase at $n = 15$ is

$$15 \times \frac{2\pi}{3} = 10\pi = 5 \times 2\pi. \tag{43}$$

Therefore

$$\mathcal{S}^{15} = \Omega^{15}\, e^{i\cdot 10\pi} = \Omega^{15}\cdot 1 = \Omega^{15} \in \mathbb{R}^+. \tag{44}$$

After five complete 2π rotations the imaginary component is exactly zero. The operator $\mathcal{S}^{15}$ is a *pure real positive scalar* with no residual phase structure to distinguish it from a plain real number. This is why the Planck mass formula in Table 1 ($\theta = 15$, $n = 0$) carries no explicit Ω factor: Ω^{15} has completed its phase life and is absorbed into the *definition* of the mass unit.

With the definitions (7.1), the Planck mass formula $M = x^{15}y^2/i$ requires the combined factor y^2/i (these are not ad hoc but occur sequentially) to supply an exact Ω^{-15} that cancels Ω^{15} from x^{15}. The cancellation is not accidental: it is the algebraic statement that Ω^{15} *is* the conversion ratio between the Ω-dimension and the Planck mass dimension. The Planck mass is Ω^{15} in natural units.

8.5 The θ-Index as a Base-15 Coordinate System

Table 1 assigns each fundamental constant a θ-index. The Ω-exponent n is recovered by

$$n = \theta - \theta_0, \tag{45}$$

where θ_0 is the nearest multiple of 15 below θ. The base-15 structure means that $\theta_0 \in \{... -30, -15, 0, 15, 30, ...\}$ partitions the constants into *octaves*, each spanning 15 units of θ. Table 3 shows the full assignment.

8.6 Re-emergence at the Electron: $\Omega^{15}\cdot\alpha^3$

Ω^{15} does not simply terminate at the Planck mass. When the Planck mass unit is cubed under the $SU(3)$ colour triplet to form the electron parameter ψ, the absorbed Ω^{15} reap-

Constant	θ	θ_0	$n = \theta - \theta_0$	n mod 3	Period
Gyromagnetic ratio	−42	−45	3	0	charge
Planck time	−30	−30	0	0	mass
Elementary charge	−27	−30	3	0	charge
Planck length	−13	−15	2	2	mass
Ampere	3	0	3	0	charge
Gravitational const. G^*	6	0	6	0	charge
Planck mass	15	15	0	0	**closure**
$\sqrt{\text{momentum}}$	16	15	1	1	—
Velocity	17	15	2	2	mass
Planck constant h^*	19	15	4	1	—
Planck temperature	20	15	5	2	atomic
Boltzmann k_B^*	29	30	−1	2	mass
Vacuum permeability μ_0^*	56	60	−4	2	mass

Table 3: θ-index decomposition for the constants of Table 1. The Ω-exponent n is the position within the current base-15 octave. Constants with $n \equiv 0 \pmod 3$ lie on the charge cycle; those with $n = 0$ at a 15-boundary sit at phase closure.

pears explicitly, coupled to the fine-structure constant α:

$$\psi = 4\pi^2\left(2^6 \cdot 3 \cdot \pi^2 \cdot \alpha_{\text{inv}} \cdot \Omega^5\right)^3 = 4\pi^2 \cdot 2^{18} \cdot 27 \cdot \pi^6 \cdot \alpha_{\text{inv}}^3 \cdot \Omega^{15} \approx 2.3896 \times 10^{22}. \tag{46}$$

The cube arises because the electron carries one unit of each of the three $SU(3)$ colour charges. Numerically,

$$\alpha_{\text{inv}}^3 \cdot \Omega^{15} = (137.036)^3 \times 34\,565.959 \approx 8.678 \times 10^8, \tag{47}$$

and $\psi = 4\pi^2 \cdot 2^{18} \cdot 27 \cdot \pi^6 \times 8.678 \times 10^8 \approx 2.390 \times 10^{22}$, consistent with CODATA.

The structural meaning is:

- Ω^{15} is the geometric amplitude of the Planck mass octave—the purely gravitational scaffold.
- α_{inv}^3 is the cube of the inverse electromagnetic coupling over the three colour charges—the matter content (note: $\alpha_{\text{inv}}^3 \gg 1$, so the coupling appears as an amplification, not a suppression, in the counting direction).
- Their product $\alpha_{\text{inv}}^3\Omega^{15}$ is the bridge between the Planck scaffold and atomic structure, first appearing at the electron.

The 6% discrepancy between the lattice CMB age (14.624 Gyr) and the observed age (13.8 Gyr) can now be given a sharper interpretation. The pure scaffold contains only the Ω^{15} gravitational term; the baryonic correction introduces the $\alpha^3\Omega^{15}$ (where $\alpha = 1/\alpha_{\text{inv}}$) electron coupling. The ratio of these two terms is $\alpha^3 = (1/137.036)^3 = (7.30 \times 10^{-3})^3 \approx 3.9 \times 10^{-7}$, far too small to account for a 6% shift directly. The correction path must therefore run through the *number of electrons per Planck mass unit*, a combinatorial factor that amplifies α^3 to the observable 6% level. Deriving this amplification factor from α alone is identified as the key outstanding problem.

8.7 The Complete Base-15 Cascade

The full logical chain from the step operator to the electron is:

$$\mathcal{S} = \Omega\, e^{i2\pi/3} \;\longrightarrow$$

$$\underbrace{\Omega^3}_{\substack{\text{charge}\\ \text{(SU3 period 3)}}} \xrightarrow{\times\,\Omega^2} \underbrace{\Omega^5}_{\substack{\text{Planck}\\ \text{temp. (period 5)}}} \xrightarrow{\times\,\Omega^{10}} \underbrace{\Omega^{15}}_{\substack{\text{phase closes}\\ \text{absorbed into } M_P}} \xrightarrow{\times\,\alpha^3\text{, cube (3 colours)}} \underbrace{\psi \;\propto\; \alpha_{\text{inv}}^3 \Omega^{15}}_{\text{electron}}. \tag{48}$$

At every stage the phase of $\mathcal{S}^n$ is $n \cdot 2\pi/3 \pmod{2\pi}$, cycling through the three classes of Table 2. The first power at which *all* cycles close is $n = 15$; this is not chosen but forced by $\text{lcm}(3,5) = 15$.

Constant	n	Physical domain	Phase class	Cycle
Gyromagnetic ratio γ_e	3	Charge	Real	Period-3
Elementary charge e^*	3	Charge	Real	Period-3
Planck length L	2	Radiation/mass	Complex	—
Ampere A	3	Charge	Real	Period-3
Gravitational constant G^*	6	Charge	Real	Period-3
Planck temperature T_P^*	5	Atomic	Complex	Period-5
Planck mass M	**0**	**Closure**	**Real**	**lcm(3,5)=15**
Planck constant h^*	4	Mixed	Complex	—
Boltzmann k_B^*	-1	Radiation	Complex	—
Vacuum permeability μ_0^*	-4	Radiation	Complex	—
Electron ψ	**15**	**Atomic+EM**	**Real**	**Re-emergence**

Table 4: Domain classification of fundamental constants by their Ω-exponent n and phase class under the base-15 cycle. The Planck mass ($n = 0$ at the closure boundary) and the electron ($n = 15$ cubed) are the two real fixed points of the cascade.

9 Macroscopic Observables: The Cosmic Microwave Background

Note on empirical input. The results in this section use a *single* empirical datum—the observed CMB black-body peak frequency $f_{\text{peak}} = 160.2$ GHz—as the sole input. All five derived quantities (age, temperature, mass density, radiation energy density, Hubble constant) follow from this one number together with the geometric scaffolding of Sections 2–6. No other observational constant is inserted.

9.1 Age and the Black-Body Peak Frequency

The Hawking temperature of a Schwarzschild singularity at Planck mass reduces to $T = T_P/(8\pi)$. Temperature resides in the Radiation Domain and therefore decays as the square root of the algorithmic age t_{age}:

$$T_{\text{cmb}} = \frac{T_P}{8\pi\sqrt{t_{\text{age}}}}. \tag{49}$$

Using the black-body peak relation ($xe^x/(e^x - 1) - 3 = 0 \Rightarrow x \approx 2.8214$),

$$f_{\text{peak}} = \frac{k_B T_{\text{cmb}}\, x}{h} = \frac{x}{8\pi^2 \sqrt{t_{\text{age}}} \cdot 2t_P}. \tag{50}$$

Setting $f_{\text{peak}} = 160.2$ GHz resolves the algorithmic age:

$$t_{\text{age}} = 0.42807 \times 10^{61} \text{ Planck time units.} \tag{51}$$

The conversion to calendar years uses the Hubble time $1/H = 2\, t_{\text{age}}\, t_P$, consistently with Eq. (55), giving $1/H = 14.624 \times 10^9$ years (+6% vs. consensus). Note that the elapsed lattice time $t_{\text{age}}\, t_P = 7.31$ Gyr is half this; the Hubble time is the appropriate age measure because the lattice radius scales as $r = 2c\, t_{\text{age}}\, t_P$, i.e. with a factor of 2 from the double-step construction. The 6% excess over the baryonic consensus age (13.8 Gyr) is expected: the model contains no baryonic matter, which would reduce the effective expansion age.

9.2 Mass Density and Dark Matter

Total cumulative mass elements scale in the Integer (circumference) Domain:

$$m_{\text{cmb}} = (2t_{\text{age}})\, m_P = 0.18636 \times 10^{54} \text{ kg}. \tag{52}$$

With volume $v_{\text{cmb}} = \frac{4}{3}\pi r^3$ and $r = 2c\, t_{\text{sec}} = 2l_p 2t_{age}$, the purely geometric mass density is

$$\rho = \frac{3m_P}{4\pi(2t_{\text{age}})^2(2l_P)^3} = 0.20998 \times 10^{-26} \text{ kg m}^{-3}. \tag{53}$$

This non-baryonic density matches the observed cold-dark-matter density $\rho_{\text{dm}} \approx 0.224 \times 10^{-26}$ kg m^{-3} to within 6%.

9.3 Radiation Energy Density

Eliminating t_{age} between the mass-domain ($\propto t^2$) and radiation-domain ($\propto \sqrt{t}$) scalings:

$$\rho_{\text{rad}} = \frac{4\sigma_{\text{SB}}}{c} T_{\text{cmb}}^4 = \frac{c^2}{1440\pi} \cdot \frac{m_{\text{cmb}}}{v_{\text{cmb}}} = 0.41716 \times 10^{-13} \text{ kg m}^{-3}. \tag{54}$$

This matches the empirical CMB radiation density to $< 0.1\%$.

9.4 Casimir Boundary

Evaluating the standard parallel-plate Casimir energy density at the radiation boundary $d_c = 2\pi\sqrt{t_{\text{age}}}$ (in Planck length units) gives $u_C = \pi^2 \hbar c/(240\, d_c^4) = 16\, u_{\text{rad}}$, where u_{rad} is the CMB radiation energy density (54). Both expressions scale as t_{age}^{-2}: $d_c \propto t_{\text{age}}^{1/2}$ gives $d_c^{-4} \propto t_{\text{age}}^{-2}$, and $T_{\text{cmb}} \propto t_{\text{age}}^{-1/2}$ gives $T^4 \propto t_{\text{age}}^{-2}$. The factor of 16 is the ratio of the parallel-plate Casimir prefactor to the Stefan–Boltzmann blackbody prefactor; it does not affect the structural identification. This algebraic equivalence maps local vacuum polarisation onto the global cosmic energy density, identifying the CMB radiation density as a global Casimir-type energy: an emergent vacuum pressure arising from the finite spatial boundary condition of the lattice.

9.5 Hubble Constant

$$H = \frac{1}{2\,t_{\mathrm{age}}\,t_P} = 0.21668 \times 10^{-17}\ \mathrm{s}^{-1} \mathrel{\widehat{=}} 66.86\ \mathrm{km\ s^{-1}Mpc^{-1}}. \tag{55}$$

Observed: 67.74 ± 0.46 km s^{-1} Mpc^{-1} (Planck 2018, 1.3% deviation).

Table 5 summarises the results.

Observable	Lattice value	Observed value	Deviation
Age (Gyr)	14.624	13.8	6%
Age (t_P units)	0.4281×10^{61}	—	—
Dark-matter density (kg m^{-3})	0.210×10^{-26}	0.224×10^{-26}	6%
Radiation energy density (J m^{-3})	0.417×10^{-13}	0.417×10^{-13}	<0.1%
CMB temperature (K)	2.7272	2.7255	0.1%
Hubble constant (km s^{-1} Mpc^{-1})	66.86	67.74	1.3%

Table 5: Lattice predictions vs. Planck 2018 observations, derived from the single empirical input $f_{\mathrm{peak}} = 160.2$ GHz.

10 The Wave-State and Point-State as the Imaginary and Real Components of the Spiral

10.1 Overview: The Oscillation is Already in the Operator

Articles 3 and 4 of this series introduce a central physical mechanism: every particle oscillates between an *electric wave-state* (extended, position undefined, duration = m_P/m Planck times) and a *mass point-state* (localised, position defined, duration = 1 Planck time). This oscillation has until now been introduced as a physical postulate. We show here that it is not a postulate at all. The wave-state and point-state are the *imaginary* and *real* components of the discrete step operator $\mathcal{S}_n = 1 + i/\sqrt{n}$, forced into existence by the same lattice construction that produces Ω.

10.2 Decomposition of the Step Operator into Two States

At each lattice step n, the operator $\mathcal{S}_n$ has two orthogonal components:

$$\mathcal{S}_n = \underbrace{1}_{\text{point-state}} + \underbrace{\frac{i}{\sqrt{n}}}_{\text{wave-state}}. \tag{56}$$

Real component (point-state). The unit real increment advances $|z_n|^2$ by exactly 1 per step, maintaining the running integer count $|z_n|^2 = n$. This is the moment of position definability: the integer domain has a fixed real value and can be assigned Cartesian co-ordinates. Its duration is 1 Planck time — one lattice step.

Imaginary component (wave-state). The term $i/\sqrt{n}$ is a purely imaginary, perpendicular rotation of magnitude $1/\sqrt{n}$. Its modulus $1/\sqrt{n}$ lives in the radiation domain and carries no fixed real projection: the imaginary axis sweeps through all directions continuously. Position is undefined. The wave-state persists for the entire interval between point-state collapses.

The correspondence with the two physical states is exact and complete:

State	Operator component	Domain	Duration
Point-state (mass)	Real: 1	Integer (n)	1 Planck time
Wave-state (electric)	Imaginary: $i/\sqrt{n}$	Radiation ($\sqrt{n}$)	m_P/m Planck times

The oscillation is therefore not an additional assumption layered on top of the lattice. It *is* the lattice, read off component by component.

10.3 Why the Wave-State Has Undefined Coordinates

In the continuous limit (Section 4),

$$z(t) = \sqrt{t}\,\left[\cos(2\sqrt{t}) + i\sin(2\sqrt{t})\right]. \tag{57}$$

This separates into amplitude and phase:

$$\text{Point-state amplitude:} \quad \sqrt{t} \quad \text{(real, growing)} \tag{58}$$

$$\text{Wave-state phase:} \quad e^{i\,2\sqrt{t}} \quad \text{(unit modulus, rotating)} \tag{59}$$

A unit-modulus complex number $e^{i\theta}$ has no preferred Cartesian projection. It is simultaneously consistent with every point on the circle of radius $\sqrt{t}$. This is not a deficiency of description; it is the correct mathematical statement that the radiation domain carries *only phase information* — no amplitude, no definite position. The wave-state has undefined co-ordinates because the imaginary axis of the spiral is, by construction, not a position but a rotation.

10.4 The Self-Referential Moment at $n = \Omega^2$

At the Planck mass domain scale $t = \Omega^2$, the step operator becomes

$$\mathcal{S}_{\Omega^2} = 1 + \frac{i}{\sqrt{\Omega^2}} = 1 + \frac{i}{\Omega}. \tag{60}$$

The wave-state imaginary component is exactly $1/\Omega$ — the *reciprocal* of the equilibrium amplitude. This is the unique self-referential moment in the lattice:

- The mass-domain amplitude is Ω (the point-state scale).
- The radiation-domain amplitude is $1/\Omega$ (the wave-state scale).
- Their product is $\Omega \cdot (1/\Omega) = 1$: the lattice unit.

No other choice of Ω preserves this unit product. The wave-state oscillation amplitude at the Planck mass scale is locked to $1/\Omega$ by the same strain-functional argument that derives Ω in Section 5. It is not tuned; it is forced.

10.5 The Electron Wave-State Duration and the Spiral

For the electron, the wave-state duration is

$$T_{\text{wave}} = \frac{m_P}{m_e} \approx 2.389 \times 10^{22} \text{ Planck time units.} \tag{61}$$

This is the electron's *intrinsic* oscillation period, measured from the particle's own point-state collapse at $n = 0$ (the start of each individual wave-state), not from the cosmic origin t_{age}. Each time the electron collapses to a point-state, the wave-state clock resets to $n = 0$ and runs for m_P/m_e Planck steps before the next collapse. The cosmic time $t_{\text{age}} \sim 10^{61}$ sets the background lattice position of the universe, but the phase accumulated *within* one electron wave-state is evaluated in the particle's own rest frame, starting fresh from $n = 0$ each cycle.

This places the electron at relative lattice coordinate $\Delta n = m_P/m_e$ at the end of each wave-state. The phase accumulated over this interval is

$$\theta_{\text{wave}} = 2\sqrt{m_P/m_e} \approx 3.09 \times 10^{11} \text{ radians.} \tag{62}$$

This phase cycles through the $SU(3)$ period $2\pi/3$ approximately

$$N_{SU(3)} = \frac{\theta_{\text{wave}}}{2\pi/3} \approx 1.48 \times 10^{11} \text{ cycles} \tag{63}$$

during one wave-state. By the colour-neutrality condition $\sum_{k=0}^{2} z_{\Omega,k} = 0$ (Section 7.2), the three colour phases average *exactly* to zero over this interval. This is the geometric explanation of why the free electron carries no net colour charge: each individual wave-state cycle is long enough to complete $\sim 10^{11}$ full $SU(3)$ rotations, averaging all colour to zero within the cycle. Colour is only defined during the point-state (1 Planck time), which is why colour-charged objects (quarks) cannot propagate freely — their point-states are too brief to establish a persistent colour state.

10.6 The Exact Spin-$\frac{1}{2}$ Result

Article 4 proposes that electron spin-$\frac{1}{2}$ arises from a half-rotation per Compton wavelength: $\omega_{\text{spin}} \cdot \lambda_e/c = \pi$. This is not an independent postulate but a direct consequence of the lattice's imaginary axis. At each Planck step the spiral phase advances by $\arctan(1/\sqrt{n}) \approx 1/\sqrt{n}$ radians. In the rest frame of the electron, the relevant lattice co-ordinate is the Compton wavelength λ_e, which spans λ_e/l_P Planck steps. Demanding that the spin phase accumulated over exactly one Compton wavelength equals π (the half-rotation defining spin-$\frac{1}{2}$) fixes the spin angular velocity to $\omega_{\text{spin}} = \pi c/\lambda_e$. The spin phase advance per single Planck time step is then simply this angular velocity multiplied by one Planck time $t_P = l_P/c$:

$$\delta\phi_{\text{spin}} = \omega_{\text{spin}} \cdot t_P = \frac{\pi c}{\lambda_e} \cdot \frac{l_P}{c} = \frac{\pi l_P}{\lambda_e}. \tag{64}$$

Since $\lambda_e = h/(m_e c)$ and $l_P = \hbar/(m_P c) = h/(2\pi m_P c)$,

$$\frac{l_P}{\lambda_e} = \frac{m_e}{2\pi m_P}, \tag{65}$$

so that $\delta\phi_{\text{spin}} = m_e/(2m_P)$. Over one complete wave-state ($T_{\text{wave}} = m_P/m_e$ steps), the total spin phase accumulated is:

$$\boxed{\Delta\phi_{\text{spin}}^{(\text{wave-state})} = \frac{m_e}{2m_P} \times \frac{m_P}{m_e} = \frac{1}{2} \text{ radian,}} \tag{66}$$

exactly, independent of all constants. This is the **geometric origin of spin-$\frac{1}{2}$**: per wave-state oscillation cycle, the spin advances precisely $\frac{1}{2}$ radian.

The connection to the ionization limit (Article 4, Section 3.4) requires careful distinction of units. Two separate facts must be kept apart:

1. **Spin rotation over 4π oscillation cycles.** Each wave-state cycle advances the spin by $\frac{1}{2}$ rad/cycle. Over 4π cycles the accumulated spin rotation is

$$\Delta\phi_{\text{spin}}^{(4\pi\text{ cycles})} = 4\pi \text{ cycles} \times \tfrac{1}{2} \text{ rad/cycle} = 2\pi \text{ rad} \quad \text{(one complete spin rotation).} \tag{67}$$

2. **Transition phase at ionisation.** The transition formula (Article 4) gives $\Phi(n) = 4\pi(1 - 1/n)$ rad, a *phase in radians* accumulated as the electron moves from shell 1 to shell n; at ionisation ($n \to \infty$) this saturates at $\Phi = 4\pi$ rad.

The self-consistency condition linking these two facts is: the 4π-radian ionisation phase coincides exactly with the oscillation count threshold (4π cycles) at which the spin completes one full 2π rotation. The shared numeral 4π is not a coincidence of notation — it reflects the same underlying geometry — but it appears in different units (radians vs. cycle count) and must not be equated directly. Below the ionisation threshold (bound states), the spin rotation over the transition is incomplete and the electron remains entangled with the nuclear potential.

Furthermore, 4π rad of transition phase corresponds to exactly 6 $SU(3)$ phase steps, since each step spans $2\pi/3$ rad:

$$\frac{4\pi \text{ rad}}{2\pi/3 \text{ rad/step}} = 6 \text{ steps} = 2 \times 3. \tag{68}$$

A free electron must therefore accumulate enough transition phase to complete *two full colour cycles* (6 $SU(3)$ steps). This is the $SU(3)$ content of the ionisation threshold.

10.7 The Electron Modulation Factor $\alpha_{\text{inv}}^3\Omega^{15}$

The base-15 cascade (Section 8) showed that Ω^{15} is absorbed into the Planck mass unit and re-emerges at the electron as

$$\psi = 4\pi^2 \cdot 2^{18} \cdot 27 \cdot \pi^6 \cdot \alpha_{\text{inv}}^3 \cdot \Omega^{15}. \tag{69}$$

The product $\alpha_{\text{inv}}^3\Omega^{15}$ has a direct interpretation in the wave-state picture:

- Ω^{15} is the *pure geometric wave amplitude* at the Planck mass closure point — the wave-state of a sterile, non-interacting geometric oscillation.
- α^3 is the electromagnetic coupling cubed over the three $\mathbb{Z}_3$ colour phases — the matter content that distinguishes the electron from a purely gravitational wave.

- Their product $\alpha_{\text{inv}}^3\Omega^{15} \approx 8.678 \times 10^8$ is the *electron's wave-state modulation factor*: the amplitude by which the electron's oscillation count exceeds the pure geometric Planck wave at the closure scale.

The electron's wave-state is not a free geometric wave but a geometric wave modulated by its electromagnetic coupling. The fact that $\alpha_{\text{inv}}^3\Omega^{15}$ is a specific, calculable number — not a free parameter — means the electron's wave-state amplitude is fixed by the same lattice that fixes Ω itself.

10.8 The Transition Spiral and the Omega Phase

Article 4 derives that the phase accumulated during an atomic transition from $n = 1$ to shell n follows the hyperbolic spiral

$$\Phi(n) = 4\pi\left(1 - \frac{1}{n}\right). \tag{70}$$

The Omega lattice phase at position $t = n^2 r_0$ is

$$\theta(n) = 2\sqrt{n^2 r_0} = 2n\sqrt{r_0}, \tag{71}$$

which is linear in n and unbounded. The transition formula $\Phi(n)$, by contrast, is bounded, saturating at 4π as $n \to \infty$. The two are not the same function; their structural connection is that both are governed by the same integer quantum number n and both encode the phase-closure condition as a boundedness constraint. In $\Phi(n)$, the bound 4π is the maximum phase before ionisation; in $\theta(n)$, the closure condition selects the integer values of n for which $\theta(n)$ is a rational multiple of 2π. The 4π limit marks the ionisation boundary in both pictures, but via different mechanisms: a saturation in Φ and a closure condition in θ. The discrete stable shells are the integer n for which $\theta(n)/(2\pi) \in \mathbb{Q}$, selecting exactly those positions on the Omega spiral where the wave-state closes coherently on itself.

The particle–wave duality of quantum mechanics is, in this picture, the projection of the spiral's complex structure onto the observable axis: measurements (point-state collapses) project the unit-modulus phase $e^{i\theta}$ onto a definite real position, producing the discrete eigenvalues of quantum mechanics, while between measurements (wave-state) the phase rotates freely with no definite real projection. There is no duality — there is only the spiral.

10.9 Summary: What Each Concept Becomes

Concept (Articles 3–4)	Omega lattice origin
Point-state (mass, defined position)	Real component of $\mathcal{S}_n$: unit increment of $\|z\|^2 = n$
Wave-state (electric, undefined position)	Imaginary component: phase rotation $e^{i\,2\sqrt{t}}$
Wave-state duration m_P/m	Lattice position $n = m_P/m$ on the Theodorus spiral
Undefined coordinates in wave-state	Unit-modulus complex number has no fixed real projection
No net colour charge of free electron	$SU(3)$ phase averages to zero over $\sim 10^{11}$ cycles in wave-state
Spin-$\frac{1}{2}$ half-rotation per λ_e	Exact: spin advance per wave-state $= \frac{1}{2}$ radian (Eq. 66)
Ionization at $\Phi = 4\pi$	4π wave-state cycles $\to 2\pi$ spin rotation + 6 $SU(3)$ steps
Electron wave-state modulation	$\alpha_{\text{inv}}^3 \Omega^{15}$: EM coupling on geometric Planck wave
Bohr integer shells (phase coherence)	Closed paths on the spiral: rational $\Phi(n)/2\pi$
Transition hyperbolic spiral $\Phi(n)$	Continuous limit of Omega lattice phase $2\sqrt{t}$

The wave-particle oscillation that underpins the entire series of articles is not an additional postulate. It is an inevitable consequence of the complex step operator $\mathcal{S}_n = 1 + i/\sqrt{n}$, which the lattice cannot avoid without collapsing to a trivial real ray and losing all physics.

11 The Imaginary Unit and the Conceptual Foundations of Quantum Mechanics

The derivations in the preceding sections were concerned with the internal logic of the Ω lattice. We now turn outward, and ask what the model implies for the outstanding conceptual problems of standard quantum mechanics (QM). Four of the deepest difficulties in the interpretation of QM — the unexplained appearance of i in the Schrödinger equation, the meaning of $|\Psi|^2$, wavefunction collapse, and the privilege of integer quantum numbers — each receive a direct geometric explanation once the imaginary unit is identified with the radiation domain of the discrete spiral.

11.1 The Origin of i in the Schrödinger Equation

Schrödinger's equation,

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi, \tag{72}$$

contains a structural mystery that Schrödinger himself acknowledged: the imaginary unit i must be present for the equation to produce oscillatory (wave) solutions rather than exponential decay. No derivation within standard QM explains *why* physical dynamics requires complex arithmetic. It is inserted by hand and justified retrospectively by agreement with experiment.

The lattice removes this mystery entirely. From Eq. (1), the universe's fundamental step operator is

$$\mathcal{S}_n = 1 + \frac{i}{\sqrt{n}}, \tag{73}$$

which is complex for every finite n. As established in Section 10, the real part governs the point-state (mass domain) and the imaginary part governs the wave-state (radiation domain). The continuous limit of this operator (Section 4) is

$$z(t) \propto \sqrt{t}\, e^{i\,2\sqrt{t}}, \tag{74}$$

which is exactly the form of a propagating wavefunction: a slowly-varying real amplitude modulated by a rapidly rotating complex phase. The Schrödinger equation (72) is the continuous differential approximation to the discrete recursion (73), obtained by taking $n \to \infty$ and replacing the finite difference with a time derivative. The i in (72) is not inserted by hand; it propagates directly from the imaginary component of $\mathcal{S}_n$.

More precisely: the trivial alternative $\mathcal{S}_n \in \mathbb{R}$ (i.e. $\phi = 0$) was excluded in Section 6.2 on the grounds that a real operator collapses the domain duality and destroys the spiral's complex geometry. A Schrödinger equation without i — which is equivalent to $\phi = 0$ — is not just empirically wrong; it is structurally forbidden by the same argument that excludes the trivial phase. The imaginary unit in quantum mechanics is a theorem of the lattice, not an assumption.

The lattice also provides a geometric account of *unitarity*. In standard QM, time evolution is governed by the unitary operator $U(t) = e^{-i\hat{H}t/\hbar}$, whose unit modulus ensures that total probability is conserved — the particle cannot spontaneously vanish. This unitarity is imposed as a postulate. In the lattice, it is automatic: the wave-state is the pure phase factor $e^{i\,2\sqrt{t}}$ of Eq. (74), which has modulus $|e^{i\,2\sqrt{t}}| = 1$ for all t. Within a single wave-state cycle, $\sqrt{t}$ changes by a fraction $\sim 10^{-39}$ (Section 11.2), so the amplitude is effectively constant and the phase factor $e^{i\,2\sqrt{t}}$ is a pure rotation that conserves it exactly. Globally, $\sqrt{t}$ grows with cosmic time, reflecting the expanding universe; unitarity is a local, per-cycle statement. It is not a postulate of the lattice; it is the statement that rotation in the complex plane does not alter the radius.

11.2 The Probability Density $|\Psi|^2$ as a Time-Average of Geometric Density

Standard QM interprets $|\Psi(\mathbf{r})|^2$ as the probability density of finding the electron at position $\mathbf{r}$ upon measurement. The electron is said to exist simultaneously at all positions, weighted by $|\Psi|^2$, until a measurement "collapses" it. This is an anti-realist interpretation: the electron has no definite position between measurements.

The lattice offers a realist alternative. The electron follows a deterministic, continuous path in the complex plane,

$$z(t) = \sqrt{t}\, e^{i\,2\sqrt{t}} = \sqrt{t}\left[\cos(2\sqrt{t}) + i\sin(2\sqrt{t})\right]. \tag{75}$$

Its real-axis (spatial) position is $\sqrt{t}\cos(2\sqrt{t})$, which oscillates rapidly through all values on the interval $[-\sqrt{t}, \sqrt{t}]$. Because the wave-state duration ($T_{\text{wave}} = m_P/m_e \approx 10^{22}$ Planck steps $\approx 10^{-22}$ s) is infinitesimal relative to any macroscopic observation time, the amplitude $\sqrt{t}$ is effectively static during a single cycle — it changes by a fraction $\Delta\sqrt{t}/\sqrt{t} \approx (m_P/m_e)/(2t) \sim 10^{-39}$ over one wave-state. The electron is therefore not smeared out in space; it is *moving through the imaginary dimension so rapidly* that its projection onto the real (measurable) axis samples all positions within a shell of effectively fixed radius $r = \sqrt{t}$.

Formally, $|\Psi(\mathbf{r})|^2$ is identified with the time-average of the geometric density of the spiral path projected onto the real (spatial) axis at position $x = \mathbf{r}$. The relevant quantity is the *real part* of $z(t)$, which sweeps the spatial axis:

$$\rho(x) = \langle \delta(\text{Re}[z(t)] - x)\rangle_{t\in[0,\,m_P/m_e]} = \left\langle \delta\Big(\sqrt{t}\cos(2\sqrt{t}) - x\Big)\right\rangle, \tag{76}$$

where the average is over one wave-state cycle of duration m_P/m_e Planck steps (Section 10.5). Because $\sqrt{t}$ is effectively constant over the cycle, this is the distribution of $r\cos\theta$ for θ uniformly distributed in $[0, 2\pi]$: the arcsine distribution $\rho(x) \propto (t - x^2)^{-1/2}$, supported on $[-\sqrt{t}, \sqrt{t}]$ and peaking at the classical turning points $x = \pm\sqrt{t}$. This correspondence with $|\Psi|^2$ is therefore structural rather than exact: the lattice produces a classical-orbit density consistent with the Bohr correspondence principle, and the precise orbital shapes of quantum mechanics require the full wave equation with its boundary conditions. The finite upper bound of the average ensures the integral is well-defined. The wavefunction's probability cloud is the time-averaged footprint of a deterministic spiral trajectory; the randomness is epistemic — coarse-graining over the wave-state duration — not ontological.

11.3 Wavefunction Collapse as Phase Alignment

The measurement problem is the most acute conceptual difficulty in QM. A particle described by a superposition $\Psi = \sum_k c_k|\psi_k\rangle$ apparently "collapses" instantaneously to a single eigenstate $|\psi_j\rangle$ upon measurement, with no dynamical mechanism. The collapse is discontinuous, apparently non-local, and has no counterpart in the Schrödinger dynamics that governs the wavefunction at all other times.

The lattice removes the need for a collapse postulate. A macroscopic measurement device is a structure whose degrees of freedom are locked to the real (Integer/Mass) domain: its state is encoded in integer-domain quantities ($n \in \mathbb{Z}^+$, $|z|^2 = n$) with no access to the imaginary radiation domain. Interaction between the device and the electron can only occur when the electron's complex phase $e^{i\,2\sqrt{t}}$ aligns with the real axis, i.e. when $\sin(2\sqrt{t}) = 0$, i.e. when $2\sqrt{t} = k\pi$ for integer k. At these moments the electron's imaginary component vanishes and it occupies a definite real-axis position: the point-state.

From Section 10.2, the point-state has duration exactly 1 Planck time. "Collapse" is therefore not a discontinuous jump but the *natural phase alignment of the spiral with the real axis*, occurring at a rate of once per m_P/m_e Planck steps. The measurement device simply selects the next naturally-occurring phase alignment and records it; the electron was already going to pass through that point-state. Nothing collapses; a pre-existing discrete event is detected.

The apparent randomness of which eigenstate is selected follows from the same time-averaging argument as Section 11.2: the device cannot know in advance which phase-alignment point it will intercept, because the alignment time depends with extreme sensitivity on the electron's precise initial conditions in the complex trajectory — conditions that are inaccessible to a mass-domain observer. The outcome is *deterministically pseudo-random*: it is generated by a fully deterministic rule (the spiral recursion $z_{n+1} = z_n \mathcal{S}_n$) but is practically unpredictable because the imaginary co-ordinate at the start of each wave-state cycle cannot be resolved by any macroscopic apparatus. This is the same structure as classical deterministic chaos — sensitive dependence on unresolvable initial conditions — not the ontological randomness that the Copenhagen interpretation requires. The Born rule emerges as the statistics of this pseudorandom process, exactly as thermodynamic probabilities emerge from deterministic molecular dynamics.

11.4 Integer Quantum Numbers as Phase-Closure Conditions

QM requires electron orbitals to carry integer quantum numbers $n = 1, 2, 3, \ldots$ because the wavefunction must be single-valued: a non-integer orbital would destructively interfere with itself on each revolution and cancel out. The integer condition is imposed as a boundary condition on Ψ but is not derived from any deeper principle within standard QM.

The lattice derives it directly. From the transition phase formula (Article 4),

$$\Phi(n) = 4\pi\left(1 - \frac{1}{n}\right), \tag{77}$$

stable orbitals are those for which $\Phi(n)$ is a rational multiple of 2π, so that the complex phase closes coherently on itself after one orbital revolution. This is the *phase-closure condition* of the discrete lattice, derived in Section 10.8 as a direct consequence of the spiral's geometry.

Integer n satisfies the closure condition exactly because

$$\frac{\Phi(n)}{2\pi} = \frac{2(n-1)}{n} \in \mathbb{Q} \quad \text{for all } n \in \mathbb{Z}^+, \tag{78}$$

which is irrational for generic non-integer n. Note that the $SU(3)$ colour-neutrality condition $\sum_k z_{\Omega,k} = \Omega\, e^{i\theta} \sum_k e^{i\,2\pi k/3} = 0$ holds for *any* θ — it is an unconditional algebraic identity of the cube roots of unity, independent of n (Section 7.2). The two conditions are therefore *separate*: phase closure selects integer n; colour neutrality is a universal property of the $SU(3)$ triplet structure. Non-integer n produces an irrational phase ratio after each revolution, which accumulates and drives the electron off the stable path via geometric strain.

The wavefunction's boundary condition (single-valuedness) is therefore the continuous differential approximation of the discrete lattice's phase-closure requirement. Integer quantum numbers are not imposed on the physics; they are selected by the geometry.

11.5 Summary: The Wavefunction as the Continuous Shadow of a Discrete Geometry

Table 6 maps each conceptual problem of QM to its geometric resolution in the Ω lattice.

The Schrödinger equation is the continuous differential equation whose solutions approximate the large-n behaviour of the discrete recursion $z_{n+1} = z_n \mathcal{S}_n$. It works because it correctly captures the dominant asymptotic structure — the amplitude $\sqrt{t}$ and the rotating phase $e^{i\,2\sqrt{t}}$ — of the spiral in the regime where n is large enough for the continuous approximation to hold. Its successes are the successes of the spiral. Its interpretational difficulties arise precisely where the continuous approximation breaks down: at the Planck scale (where the discrete structure is unresolved), at the point of measurement (where the continuous wave-state must yield to the discrete point-state), and at the quantum number boundaries (where the lattice's exact integer structure is replaced by a boundary condition on a smooth function).

The imaginary unit is not a mathematical convenience inserted into physics from the outside. It is the structural signature of the radiation domain, present in the lattice from the first step, and necessary — as Section 6.2 proves — for the existence of distinct mass and radiation domains at all. Quantum mechanics works because the universe is built on a complex geometry. The wavefunction is not the fundamental object; the spiral is.

12 Conclusion

We have derived $\Omega = \sqrt{\pi^e e^{1-e}}$ from first principles, closing the logical gaps that remained in earlier work. The key steps are:

1. **The spiral's step operator $\mathcal{S}_n = 1 + i/\sqrt{n}$ is intrinsically complex**, encoding the integer/radical domain duality from the outset. The continuous limit $z(t) \propto \sqrt{t}\, e^{i\,2\sqrt{t}}$ is exact in the large-t regime.

2. **The capacity functional** $F(x) = e \cdot (\pi/x)^x$ has a geometric *maximum* at $x = \pi/e$ (maximum structural efficiency) and a natural e-fold boundary at $x = e$ (where $\ln x = 1$, one unit of logarithmic action). Both properties belong to the same function; their incommensurability is the lattice's inherent tension between structural efficiency and self-reference.

3. **Evaluating F at the e-fold boundary** $x = e$ yields $\Omega^2 = F(e) = e \cdot (\pi/e)^e = \pi^e e^{1-e}$ directly. The factor $e^{e-1} = e^e/e$ is the self-referential cost x^x at $x = e$ divided by the primitive unit e; no additional assertion is required.

4. **The trivial phase $\phi = 0$ is excluded** because a real global operator collapses the domain duality, annihilates the spiral's complex rotation, and renders mass and charge structurally indistinguishable.

5. **The non-trivial phase locks to** $\phi = 2\pi/3$, generating the cyclic group $\mathbb{Z}_3$—the discrete centre of $SU(3)$—and the fractional $-1/3$, $+2/3$ quark charges, without requiring the full 8-dimensional $SU(3)$ Lie algebra.

6. **The complex position** $z_\Omega = \Omega\, e^{i\,2\Omega}$ on the spiral is self-referential—the phase 2Ω is determined by Ω itself—and the three $SU(3)$ partners $z_{\Omega,k} = \Omega\, e^{i(2\Omega + 2\pi k/3)}$ satisfy $\sum_k z_{\Omega,k} = 0$, the colour-neutrality condition.

7. **The base-15 phase closure** (Section 8) is the central new result. The $SU(3)$ charge cycle has period 3 and the atomic/temperature cycle has period 5; their least common multiple $\mathrm{lcm}(3, 5) = 15$ forces the first simultaneous closure at Ω^{15}. At this

point the accumulated phase is 10π—exactly five full rotations—so $\mathcal{S}^{15} = \Omega^{15}$ is purely real with no residual phase tag. Ω^{15} is therefore absorbed into the definition of the Planck mass unit, explaining why the Planck mass formula in Table 1 carries no explicit Ω factor.

8. **Ω^{15} re-emerges at the electron.** When the Planck mass unit is cubed under the $SU(3)$ colour triplet, the absorbed Ω^{15} reappears explicitly in the electron parameter $\psi = 4\pi^2 \cdot 2^{18} \cdot 27 \cdot \pi^6 \cdot \alpha_{\text{inv}}^3 \cdot \Omega^{15}$. The product $\alpha_{\text{inv}}^3 \Omega^{15}$ is the bridge between the purely gravitational Planck scaffold and atomic structure.

9. **The θ-index is a base-15 coordinate.** Every fundamental constant's Ω-exponent $n = \theta - \theta_0$ (where θ_0 is the nearest base-15 boundary below θ) gives its position within the current phase octave. Constants with $n \equiv 0 \pmod 3$ are real (charge cycle); $n = 0$ at a 15-boundary signals full phase closure.

10. **Using $f_{\text{peak}} = 160.2$ GHz as the sole empirical input**, the lattice reproduces five independent CMB observables to within 6%. The residual is attributable to the absence of the baryonic $\alpha_{\text{inv}}^3 \Omega^{15}$ electron coupling in the pure Planck scaffold; deriving the amplification factor that converts α^3 to an observable 6% correction is identified as the key outstanding problem.

11. **The wave-state and point-state are the imaginary and real components of $\mathcal{S}_n$** (Section 10). The real unit increment is the point-state (1 Planck time, defined position); the imaginary phase rotation $e^{i\,2\sqrt{t}}$ is the wave-state (duration m_P/m, undefined position). This oscillation is not a postulate — it is the operator, decomposed.

12. **Spin-$\frac{1}{2}$ is exact and parameter-free.** The spin phase advance per wave-state cycle evaluates to $\frac{1}{2}$ radian exactly (Eq. 66), independent of all constants. The 4π ionization threshold is the minimum phase for the spin to complete one full 2π rotation, and it contains exactly 6 $SU(3)$ steps — two complete colour cycles — the structural condition for a free electron.

13. **The electron modulation factor $\alpha_{\text{inv}}^3 \Omega^{15}$** is the ratio of the electromagnetic coupling to the pure geometric wave amplitude at the Planck closure point. It bridges the gravitational scaffold to atomic structure and is fully determined by α and the lattice.

The Unified Picture: Tying the Series Together

The seven articles of this series each illuminate a different face of a single underlying structure: the Spiral of Theodorus in the complex plane, stepped by $\mathcal{S}_n = 1 + i/\sqrt{n}$, running from the Planck scale to the cosmic scale. Table 7 shows how the series fits together.

The result is a logically complete, parameter-free derivation of Ω from the Spiral of Theodorus in which every major feature — domain duality, $\mathbb{Z}_3$ phase symmetry, base-15 closure, wave-state oscillation, spin-$\frac{1}{2}$, and the CMB observables — is a mathematical consequence of the operator $\mathcal{S}_n = 1 + i/\sqrt{n}$ and the single equilibrium condition that produces Ω. No other value of Ω and no other oscillation structure is consistent with the combined constraints. The closure scale 15 is forced by $\text{lcm}(3,5) = 15$, where period 3 is the $SU(3)$ charge cycle (Section 6) and period 5 is read from the Ω-exponent of the Planck temperature in the base-15 table; deriving period 5 from first principles is identified as an open problem for future work. The wave-particle duality of quantum mechanics, usually introduced as an axiom, is here a theorem: there is no complex spiral without both a real amplitude (point-state) and an imaginary phase (wave-state), and there is no lattice without the spiral.

A The Role of α in the Lattice: Scale and Status

Overview

The fine-structure constant $\alpha \approx 1/137.036$ does not appear in the Planck-scale lattice geometry. It enters at the Compton scale, where it governs how the lattice connects the Planck step to the atomic orbital. This appendix documents the four distinct levels at which α appears, based on the geometric models of atomic orbitals (Article 4) and the w-axis synthesis (Article 5). The Python programme that accompanies this article (available at `http://codingthecosmos.com/Theodorus-Omega-base15.py`) illustrates the Planck-scale behaviour described in Sections 2 through 8; α does not appear anywhere in that code.

A.1 Layer 0: The Planck Scale — α is Absent

The Theodorus step operator $\mathcal{S}_n = 1 + i/\sqrt{n}$ contains no α. The capacity functional $F(x) = e \cdot (\pi/x)^x$ contains no α. The MLTA objects $M = 1$, $T = \pi$, $P = \Omega$ contain no α. Alpha enters the MLTA framework at exactly one place: the Ampere object

$$A = \frac{2^7 \pi^3 \Omega^3}{\alpha_{\text{inv}}},$$

which is the only geometrical object that encodes electromagnetism. Every other MLTA object is α-free. This is not an accident: the Planck scale runs on π and Ω alone. Alpha is not a Planck-scale constant.

A.2 Layer 1: The Compton Scale — α as the Unit-Step Resolver

Alpha first appears at the bridge between the Planck step and the atomic orbital. The Compton wavelength $\lambda_e = h/(m_e c)$ is the oscillation step unit for the electron. Article 4 establishes the following α-dependent quantities at this scale:

$$r_{\text{incr}} = \frac{1}{2\pi \cdot 2\alpha_{\text{inv}}}, \qquad \text{(radial change per Compton step)} \qquad (79)$$

$$r_\alpha = \sqrt{2\alpha_{\text{inv}}} = \sqrt{2/\alpha}, \qquad \text{(angular scale per Compton step)} \qquad (80)$$

These are not definitions of α but consequences of the question "how much does the orbital radius change per quantum of action?" Alpha is the answer: it is the ratio of the electromagnetic interaction energy to the quantum of mechanical action, which is why $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ in standard notation.

A.3 Layer 2: The Orbital Scale — α Sets the Scale, Not the Shape

The quantization condition that forces discrete energy levels is

$$\Phi(n) = 4\pi \left(1 - \frac{1}{n}\right), \tag{81}$$

which contains *no* α. The topology of quantization — the reason energy levels are discrete, the reason the electron must land on integer n, the ionization bound at $\Phi \to 4\pi$ — is purely geometric.

Alpha determines *scale and timescale*:

$$r_0 = 2\alpha_{\text{inv}}\lambda_e \qquad \text{(Bohr radius)} \tag{82}$$

$$T_1 = 2\pi\,(2\alpha_{\text{inv}})^2 = 471\,964 \text{ Compton steps} \qquad \text{(orbital period at } n{=}1\text{)} \tag{83}$$

but not the shape of the spiral. The observation that $\Phi(n)$ is α-free is the clearest evidence that α is a scale constant at the atomic level, not a topological one.

A.4 Layer 3: The W-Axis — α as the Inter-Domain Bridge Probability

Article 5 provides the deepest picture. The difference between gravitational and atomic orbital periods is not a free parameter:

$$T_{\text{grav}} \propto r_\alpha \qquad \text{(one coincidence: mass domain only)} \tag{84}$$

$$T_{\text{atom}} \propto r_\alpha^2 \qquad \text{(two coincidences: mass domain AND w-axis)} \tag{85}$$

The w-axis (charge domain) gate opens with probability $1/r_\alpha$ per Planck step. An atomic orbital transition requires both the mass-domain event and the w-axis event simultaneously, so the joint probability is $1/r_\alpha^2$ and the expected waiting time is $r_\alpha^2 = 2/\alpha$.

This is the deepest reframing of α: it is the *opening probability of the charge-domain gate per Planck step.* It is not embedded in the step operator $\mathcal{S}_n$ (which runs at the Planck rate regardless of α), but governs how often the electromagnetic channel is accessible.

A.5 The $\sqrt{2\alpha}$ Appearance: Why the Factor of 2

Throughout Article 4, $r_\alpha = \sqrt{2\alpha_{\text{inv}}}$ rather than $\sqrt{\alpha_{\text{inv}}}$ appears. The factor of 2 is not an ad hoc normalisation. Article 5's three-wave model identifies two mass-domain waves (Waves 1 and 2 in the xy-plane) that must coincide before the w-axis wave (Wave 3) becomes accessible. The rate of the two-wave coincidence is $(1/r_\alpha)^2 = 1/(2\alpha_{\text{inv}})$; the single-wave rate is therefore $1/r_\alpha = 1/\sqrt{2\alpha_{\text{inv}}}$. The factor of 2 is the number of mass-domain waves, not a free parameter. This is why $\beta = \sqrt{2\alpha}$ is the natural electromagnetic bridge unit, rather than $\sqrt{\alpha}$.

A.6 The α-Free Core: $\Phi(n) = 4\pi(1 - 1/n)$

This formula determines which orbital radii are stable. It closes at 4π (the ionization limit, two full rotations). Integer n values occur at rational multiples of 2π. The complete discrete structure of atomic energy levels follows from this formula and from π alone. Alpha provides the physical ruler that converts dimensionless windings into wavelengths and frequencies, but it does not determine which windings are allowed.

A.7 Summary: What the Lattice Does and Does Not Give for α

What the lattice provides for α	What the lattice does not provide
Scale at which α first appears (Compton, not Planck)	Why $r_\alpha \approx 16.555$ rather than another value
Physical meaning: $\alpha = 2/r_\alpha^2$ as w-axis gate probability	A derivation of α from π and e alone
Factorisation: $\beta = \sqrt{2\alpha}$, the two-wave bridge unit	
Beta closure $\beta^6 = (2\alpha)^3$ at $\mathrm{lcm}(2,3) = 6$	
Topological identity $\Phi(n) = 4\pi(1 - 1/n)$, fully α-free	

The correct scientific position is explicit: α is the one free parameter of the electromagnetic sector, entering at the Compton-to-orbital bridge. Everything above (the Planck scale, Ω, the base-15 cascade to ψ) is determined by geometry; everything at and below the atomic scale is governed by α as a given coupling strength. In the language of the simulation hypothesis that motivates the series: α is embedded in the source code. The lattice geometry explains *where* it appears and *what it does*; deriving its numerical value from first principles is identified as the central open problem of the series.

B Reference Programme

The Python programme `Theodorus-Omega-base15.py` is the computational reference for this article. It is available at:

`http://codingthecosmos.com/Theodorus-Omega-base15.py`

The programme implements the incremental Theodorus lattice (Planck steps $n = 1 \ldots 25$), the τ-loop bridge $A(\tau) = \Omega^{\ln \tau + 1}$, and the complete base-15 phase cascade $k = 0 \ldots 15$. Six panels are generated:

1. **3D spiral**: Theodorus geometry, colour-coded by $k_{\text{frac}} = \ln \tau + 1$.
2. **XY projection**: top-down view, colour-coded by wave amplitude.
3. Ω_n **convergence**: Ω_n vs. step n, showing how Ω emerges as π_n and e_n converge.
4. **Phase wheel**: $\mathcal{S}^k = \Omega^k e^{ik\cdot 2\pi/3}$ in the complex plane; period-3 (charge) and period-5 (temperature) cycles marked.
5. **Amplitude bar**: $\log_{10}(\Omega^k)$ for $k = 0 \ldots 15$; $\Omega^{15} = 34\,565.959$ at the closure boundary.
6. τ**-loop bridge**: $A(\tau)$ sweeping from k=1 (radiation bridge) to k=2 (matter domain) within one Planck step.

The programme uses the Nilakantha series for π (error $< 0.001\%$ at $n = 25$, matching the spiral step count so both coordinates share the same index n) and requires only the Python standard library plus NumPy and Matplotlib. Alpha does not appear anywhere in the code.

References

[1] M. J. Macleod, *Programming Planck units from a virtual electron; a Simulation Hypothesis*, Eur. Phys. J. Plus **133**, 278 (2018).

[2] M. J. Macleod, *Do these anomalies in the physical constants constitute evidence of coding?* `https://doi.org/10.2139/ssrn.4346640`

[3] Planck Collaboration, *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. **641**, A6 (2020). arXiv:1807.06209.

[4] D. J. Fixsen *et al. The Cosmic Microwave Background temperature spectrum from the full COBE FIRAS data set*, Astrophys. J. **473**, 576 (1996).

[5] J. Barrow and D. J. Shaw, *The Value of the Cosmological Constant*, arXiv:1105.3105 [gr-qc] (2011).

[6] M. J. Macleod, *4. Geometrical origins of quantization in H atom electron transitions (a Simulation Hypothesis model)*, `https://doi.org/10.2139/ssrn.3703266`

[7] M. J. Macleod, *5. W-Axis Synthesis: Dimensional Momentum and the Unified Planck Scale*, `https://doi.org/10.13140/RG.2.2.10680.20487/1`

[8] M. J. Macleod, `Theodorus-Omega-base15.py` — Reference programme for this article, `http://codingthecosmos.com/Theodorus-Omega-base15.py`

QM concept / mystery	**Standard interpretation**	**Lattice resolution**
i in Schrödinger eq.	Inserted by hand; no physical explanation	Imaginary component of $\mathcal{S}_n = 1 + i/\sqrt{n}$; excluded from real-only ($\phi = 0$) by Section 6.2
$\|\Psi\|^2$ as probability	Anti-realist: no position between measurements	Time-average of geometric density of the deterministic spiral path over the real axis; Eq. (76)
Wavefunction collapse	Discontinuous, non-local, no mechanism	Phase alignment of $e^{i\,2\sqrt{t}}$ with real axis; occurs naturally every m_P/m_e Planck steps; duration 1 Planck time
Integer quantum numbers	Boundary condition on Ψ; not derived	Phase-closure condition of the discrete lattice; Eq. (78); $\Phi(n)/(2\pi) \in \mathbb{Q}$ iff $n \in \mathbb{Z}^+$ (colour neutrality is universal, separate)
Wave–particle duality	Axiomatic; electron is "both at once"	Real part of $z(t)$: point-state (particle); imaginary part: wave-state; operator $\mathcal{S}_n$ has both by construction
Origin of $\hbar$	Fundamental constant; unexplained	Encodes the ratio of the Planck action unit to the lattice step; emerges from the product $m_P l_P c$ at the phase-closure scale Ω^{15}

Table 6: Resolution of six foundational conceptual problems of quantum mechanics by the Ω lattice. Each mystery arises from treating the continuous wavefunction as fundamental rather than as the large-n limit of the discrete complex spiral $\mathcal{S}_n = 1 + i/\sqrt{n}$.

Art.	Subject	Key result	Omega-lattice origin
1	Planck-scale CMB	CMB observables from single input f_{peak}	Spiral radius $\sqrt{t}$ = radiation domain; circumference $2\pi\sqrt{t}$ = mass domain
2	Ω derivation (this article)	$\Omega = \sqrt{\pi^e e^{1-e}}$ forced by two incommensurable constraints	Capacity functional $F(x) = e \cdot (\pi/x)^x$; maximum at $x = \pi/e$, e-fold boundary at $x = e$
3	Planck-scale orbitals	Atomic shells from Planck-unit geometry	Integer closures $\lvert z_n\rvert^2 = n$ select stable orbital radii
4	Two-photon model	Spin-$\frac{1}{2}$, ionization at 4π, hyperbolic transition spiral	Phase $2\sqrt{t}$; spin $= \frac{1}{2}$ rad/cycle; $4\pi = 6$ $SU(3)$ steps
5	(forthcoming)	Proton mass from α	$\alpha^n\Omega^m$ bridge beyond the electron
6	(forthcoming)	Strong force and confinement	$SU(3)$ triplet $z_{\Omega,k}$; colour sum $= 0$
7	(forthcoming)	Cosmological constant	Casimir boundary at $d_c = 2\pi\sqrt{t_{\text{age}}}$

Table 7: Map of the seven-article series onto the Ω lattice. Every result in the series is a consequence of the single complex step operator $\mathcal{S}_n = 1 + i/\sqrt{n}$ and the equilibrium constant $\Omega = \sqrt{\pi^e e^{1-e}}$.