

8. Holographic Emergence in the Simulation Hypothesis: The Live Shell, its Thickness, and Area Scaling

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Final article of the “Mathematical Electron” series

Abstract

If holographic area scaling is a fundamental constraint on quantum gravity, a Planck-scale model should either reproduce it or explain why it does not apply. With the model of this series now complete, the question can be answered precisely. The model is holographic in three definite senses. First, the interior of the expanding hypersphere carries no independent degrees of freedom: it is the past, determined by the present state and an invertible update rule. Second, the live universe is a shell. A massive particle is defined as a point only once in every $\psi = m_P/m$ Planck ticks, and during each wave-state interval the surface advances radially by $k\psi l_P = k\bar{\lambda}$, so each species occupies a shell whose radial thickness is k times its reduced Compton wavelength, where k is the prefactor of the radius law $R = kct$. Third, gravitational pairing between point-states scales with area: a mass M carries M/m_P point-states per tick and so $\frac{1}{2}(M/m_P)^2$ pairs, which is exactly the Bekenstein–Hawking entropy of a black hole of the same mass divided by 8π . The universe’s mass grows in proportion to its radius, the defining relation of a black hole, so its pairing count is of the order of the holographic bound of the Hubble horizon, $\sim 10^{122}$. The model does not claim that three-dimensional space is reconstructed from a two-dimensional surface: its surface is three-dimensional, and its thickness lies in the fourth, radial direction.

1 Introduction

The “Mathematical Electron” (Programmer God) series [1] proposes that physical reality is generated from a small set of dimensionless geometric objects. The principal physical input is the fine-structure constant α , supplemented by π and

$$\Omega = \sqrt{\pi^e e^{(1-e)}}. \quad (1)$$

This article comes last because its subject, the sense in which the model is holographic, depends on the rest of the series: the τ -loop architecture and duty cycle of Article 1b [3], the hypersphere and its relativity in Article 2 [4], gravitational pairing between point-states in Article 3 [5], and the atomic model of Article 4 [6]. An earlier draft collected ideas before that model was complete; this version rebuilds them on it.

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Standard holographic principles relate a higher-dimensional gravitational description to lower-dimensional boundary data, most clearly in AdS/CFT [10], and the holographic bound [11, 12, 13] limits the information in a region by the area of its boundary. The present model is not an AdS/CFT duality. It is holographic in three definite senses, developed in Sections 4–6:

- (i) **Interior.** The interior of the hypersphere carries no independent degrees of freedom.
- (ii) **The live shell.** The universe being computed at any tick is a shell of finite radial thickness, set for each massive species by its reduced Compton wavelength.
- (iii) **Area scaling.** Gravitational pairing between point-states scales with area, with the same mass dependence as the Bekenstein–Hawking entropy.

Sections 2 and 3 set out the data layer and the geometry on which these rest, and Section 7 compares the result with standard holographic frameworks. The surface of the model is three-dimensional; its thickness lies in the fourth, radial direction.

2 The dimensionless data layer

2.1 The two domains

The update rule of the Planck scaffolding is the Theodorus step

$$z_{n+1} = z_n S_n, \quad S_n = 1 + \frac{i}{\sqrt{n}} \quad (2)$$

(Article 1b). Its state is a single complex number, and its two real components are the two domains of the model:

$$|z_n|^2 = n \quad (\text{Matter, or Integer, Domain}), \quad (3)$$

$$\arg z_n = \Theta_n \approx 2\sqrt{n} \quad (\text{Radiation, or } \sqrt{\text{Integer}}, \text{ Domain}). \quad (4)$$

The Matter Domain carries the objects M , L , T , V and grows linearly with the step count; the Radiation Domain carries the phase, and with it the ampere A and the square-root momentum P , and grows as $\sqrt{n}$. In Article 2 the first is the h -axis of the hypersphere, its radius, and the second is the w -axis of Article 5 [7], a phase domain rather than a direction in space. The update state is therefore two-dimensional in the sense of having two domains, not in the sense of a two-dimensional spatial surface.

2.2 Dimensioned and dimensionless

The objects M , T , P , ... are dimensionless geometric forms. They acquire physical dimension only through the scalars r and v that translate them into SI units (Article 6 [8]), and in the render invariant of Article 1b the scalars cancel altogether:

$$\frac{P^{15} T^2}{M^{12}} = \pi^2 \Omega^{15} \equiv I. \quad (5)$$

The same quantity can therefore be read either as a combination of dimensioned units or as a pure number, depending on which side of the equation is used. I is the volume of the rotation group $SO(3)$ at radius Ω^5 , and the state space in which each tick is held is the 3-sphere, of volume $2I$ (Article 1b). The data layer of the model is dimensionless; dimension is a property of how it is read out.

3 The surface and its radius

3.1 The three-sphere

Observable space is the three-dimensional surface of a four-dimensional hypersphere of radius $R(t)$ (Article 2). Its metric,

$$d\Sigma_3^2 = R^2(t) [d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2)], \quad (6)$$

can be written, with $u = R(t)\chi$ on a slice of fixed cosmic time, as the warped foliation

$$d\Sigma_3^2 = du^2 + R^2(t) \sin^2 \left(\frac{u}{R(t)} \right) d\Omega_2^2, \quad d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2, \quad (7)$$

a one-parameter family of two-sphere screens. The largest screen, at $\chi = \pi/2$, has area

$$A_{\text{eq}} = 4\pi R^2(t). \quad (8)$$

All three coordinates (χ, θ, ϕ) are tangent to the surface. The radius is normal to it.

3.2 The radius is cosmic time, and it is observable

The radius is the h -axis of Article 2: it measures the expansion and is shared by everything on the surface. Relativity lives in a different plane, that of lateral motion and proper time, in which every object moves at c , and the radius does not enter it (Article 2, Appendix 2). We write the radius law as

$$R(t) = k c t, \quad (9)$$

with k a pure number set by the radius law of the series (Articles 1 and 2).

In Planck units $R/l_P = k t_{\text{age}}$ is a pure number, as the dimensionless data layer suggests. A dimensionless radius is not, however, an unobservable one. The curvature of a three-sphere is intrinsic to it and can be measured from within. By the null condition of Article 2, light emitted at redshift z has travelled an angle

$$\chi = \int \frac{c dt}{R} = \frac{\ln(1+z)}{k} \quad (10)$$

around the hypersphere. For $k = 1$ the antipode lies at $z \approx 22$, light from the microwave background ($z \approx 1100$) would have travelled more than once around the universe, and space would be strongly curved, with $\Omega_k = -1/k^2 = -1$. The near-flatness of observed space requires $k \gg 1$. The value $k = 4\pi$ stated in Article 1 [2] gives $\Omega_k \approx -0.006$, close to present bounds, which are themselves derived within the standard cosmological model. The analysis below is written for general k .

4 The live shell and its thickness

4.1 When a particle is defined

A massive particle is a point only intermittently. It occupies the mass point-state for one Planck tick in every

$$\psi = \frac{m_P}{m}, \quad (11)$$

and between point-states it is a wave-state with no coordinates (Article 1b). Its mass is the frequency of its point-states, $m = m_P/\psi$ (Article 3). For the electron $\psi_e \approx 2.39 \times 10^{22}$ (Articles 1b and 7 [9]).

4.2 The thickness

During one wave-state interval, of ψ ticks, the surface advances radially by

$$\Delta h = k \psi l_P = k \bar{\lambda}, \quad (12)$$

since $\psi l_P = \hbar/(mc) = \bar{\lambda}$ is the particle's reduced Compton wavelength. (The identity holds exactly with the model's Planck length; with the CODATA value it holds to 130 ppm, the same offset between model and CODATA Planck units that appears in Article 5.) Each massive species therefore occupies a live shell of radial thickness $k\bar{\lambda}$:

species	$\psi = m_P/m$	$\bar{\lambda}$	shell thickness
electron	2.39×10^{22}	$3.86 \times 10^{-13} \text{ m}$	$k\bar{\lambda}_e$
proton	1.30×10^{19}	$2.10 \times 10^{-16} \text{ m}$	$k\bar{\lambda}_p = k\bar{\lambda}_e/1836$
neutrino (0.05 eV)	2.4×10^{29}	$3.9 \times 10^{-6} \text{ m}$	$k\bar{\lambda}_\nu$

The lightest massive species set the radial depth of the live universe. For $k = 4\pi$ the electron's shell is exactly two full Compton wavelengths thick, $4\pi\bar{\lambda}_e = 2\lambda_e$.

4.3 What the thickness is not

The thickness is radial: it records *when* a particle is defined, once in every ψ ticks, not *where*. It is not the probability cloud of the particle. That cloud lies in the surface, among the possible locations of the next point-state. For the electron in hydrogen its size is set by the orbit, $r_1 = 2a_\mu \approx 274\bar{\lambda}_e$ in the model (Article 4), and its orientation by the N-S axis, which selects each collapse point (Article 2). The radial thickness is smaller than the cloud — some 270 times for $k = 1$, and 22 times for $k = 4\pi$ — and, more to the point, lies in a different direction.

For $k = 1$ the thickness equals $\bar{\lambda}$ exactly, but that equality is the relation $\lambda = cT$ between a wavelength and its period, taken at the speed of expansion. It identifies the thickness; it does not derive the Compton wavelength.

4.4 Photons

A photon never reaches a point-state (Article 1b). It has no proper time and occupies no shell thickness: it rides the surface outward and propagates across it at c . Its wavelength is lateral, and it stretches in proportion to R (Article 2, Appendix 2), belonging to the plane of lateral motion alone.

5 The interior carries no independent degrees of freedom

The interior of the hypersphere, at radii smaller than $R(t)$, is the past. The step rule is algebraically invertible (Article 1b), so the past is determined by the present state together with the rule and adds no independent degrees of freedom. Whether it is held as frozen shells (Article 1b, Appendix A) or recomputed, it is not an independent store of information. The information of the universe is carried by the live shell. This is the sense of the comparison Article 2 draws between the hypersphere and a black hole, whose information is likewise held on its surface; Section 6 makes the comparison quantitative.

6 Area scaling from point-state pairing

6.1 The pairing count of a mass

Gravitational orbital pairs form between particles in the point-state (Article 3). A body of mass M , whatever its composition, contains M/m_P point-states per tick, so the number of point-state pairs it carries per tick is

$$N_{\text{pair}} = \frac{1}{2} \left(\frac{M}{m_P} \right)^2. \quad (13)$$

6.2 Comparison with Bekenstein–Hawking

A black hole of mass M has horizon area $A = 4\pi R_s^2$, with $R_s = 2GM/c^2$, and entropy

$$\frac{S_{\text{BH}}}{k_B} = \frac{A}{4l_P^2} = 4\pi \left(\frac{M}{m_P} \right)^2. \quad (14)$$

Therefore

$$\boxed{N_{\text{pair}} = \frac{S_{\text{BH}}}{8\pi k_B}} \quad (15)$$

for every mass. The pairing count scales exactly as the Bekenstein–Hawking entropy — as the square of the mass, which is the area of the horizon — and differs from it by the fixed factor 8π . The result uses only the duty cycle and the pairing rule, and it is independent of k . If each point-state pair carries relational information of order one, the information capacity of a gravitating mass scales as the area of its horizon, not the volume it encloses.

The factor 8π is not derived. The number of pairs is an upper count of relational data: phase coherence, conservation rules and geometric redundancy correlate pairs, and the effective number of independent degrees of freedom is smaller,

$$\frac{S}{k_B} \sim \eta N_{\text{pair}}, \quad 0 < \eta \leq O(1). \quad (16)$$

The scaling is unaffected; the coefficient is not claimed.

6.3 The universe

In the model the universe’s mass grows linearly with the step count, as does its radius (Articles 1 and 2): its mass is proportional to its radius, which is the defining relation of a black hole. This is the model’s form of a known observation, that the Hubble sphere lies at the Schwarzschild radius of its own mass. For $H = 1/t_{\text{age}}$ the Hubble-sphere mass is $M_H = c^3/(2GH)$, whose Schwarzschild radius $2GM_H/c^2$ equals the Hubble radius c/H exactly, with $M_H/m_P = t_{\text{age}}/2$. With $t_{\text{age}} \approx 8.1 \times 10^{60}$ Planck times,

$$N_{\text{pair}} \approx 8 \times 10^{120}, \quad \frac{S_{\text{BH}}(\text{Hubble})}{k_B} = \pi t_{\text{age}}^2 \approx 2 \times 10^{122}, \quad (17)$$

in the ratio 8π again. The pairing count of the universe is thus of the order of the holographic bound of its horizon — the 10^{122} of the cosmological holographic bound — and it grows as area, t_{age}^2 , rather than as volume, t_{age}^3 .

7 Comparison with holographic frameworks

The holography of this model is structural and algorithmic, not a proven duality. Table 1 summarizes the distinction.

Table 1: Standard holographic frameworks and the present model

Feature	Standard frameworks, especially AdS/CFT	This model
Fundamental theory	Boundary quantum field theory / string-theoretic construction	Discrete geometric algorithm on a dimensionless data layer
Reconstruction parameter	Energy scale, RG flow, or radial coordinate	The radius $R = kct$, the h -axis: cosmic time
Holographic content	Duality between bulk and boundary descriptions	Interior carries no independent degrees of freedom; live shell of thickness $k\bar{\lambda}$; pairing count scaling as area
Gravity mechanism	Gauge/gravity duality and entanglement structure	Pairing of point-states; weakness from their rarity (Article 3)
Entropy scaling	$S = A/4l_P^2$	$N_{\text{pair}} = S_{\text{BH}}/8\pi$ for every mass; coefficient not derived
Curvature	Often negative (AdS)	Positive and closed; near-flat for $k \gg 1$
Time	Boundary time related to bulk time by the duality	Cosmic time is the radius; proper time is each particle's own clock

The most significant divergence is ontological. Standard holography is usually formulated as a duality between theories. The present model proposes a pipeline from code to geometry. It does not require conformal field theories, large- N limits, or negative curvature, and it is therefore aimed at a different problem: how an expanding, nearly flat universe could arise from dimensionless Planck-scale update data. This places it closer in spirit to general holographic bounds and screen-based ideas than to a specific AdS/CFT construction [13].

7.1 An inverse dictionary

With the model complete, several effective observables can be traced back to update-layer quantities:

- **Mass** is the frequency of point-states, $m = m_P/\psi$ (Articles 1b and 3).
- **The gravitational coupling** $\alpha_G = (m_e/m_P)^2$ is the rate of simultaneous point-states, and the equivalence principle follows from the point-state rate being proportional to mass (Article 3).
- **Charge** is the power of T needed for scalar cancellation (Articles 5 and 7).
- **The atomic spectrum** is the reduced-mass Rydberg formula exactly, with $L = n\hbar$ and phase $\Phi(n) = 4\pi(1 - 1/n)$ derived from the rotation law (Article 4). The integer n is assumed rather than derived, and the fine structure, a function of α alone, is the model's next target.
- **Orbital period and precession** map to alignment ratios and the coefficient k_r of the pairing simulations (Article 3); the precession relation is still taken from fits.

This is not a full holographic dictionary in the AdS/CFT sense, which would require a systematic mapping of states, operators, correlation functions, entropy and dynamics. It does show that reconstruction runs in both directions.

7.2 Prospect: horizons

In classical general relativity an event horizon is a null surface. In the present model a horizon can be treated as a reconstruction interface, at which incoming wave-point processes are sampled over a finite wave-state interval. Equation (15) shows that the pairing count of a black hole already carries the scaling of its entropy, so the information accounting has the right form. This is not a derivation of Hawking radiation or a resolution of the information paradox. Hawking-like emission would correspond to the release of pairing data through the reconstruction pipeline, and the emission spectrum, the unitarity map and any Page-curve behaviour remain future work.

8 Conclusion

The model of this series is holographic in three definite senses. The interior of the hypersphere is the past, determined by the present state and an invertible rule, and carries no independent degrees of freedom. The live universe is a shell: a massive particle is defined only once in every $\psi = m_P/m$ ticks, so each species occupies a radial thickness $k\bar{\lambda}$, a statement about when a particle is defined rather than where. And gravitational pairing between point-states scales as area: for every mass, the pairing count is the Bekenstein–Hawking entropy divided by 8π , and for the universe, whose mass tracks its radius, it reaches the 10^{122} of the cosmological holographic bound.

Three questions remain open. The factor 8π between the pairing count and the Bekenstein–Hawking entropy is not derived. The radius prefactor k , on which the shell thickness depends, must be fixed consistently across the series, and the near-flatness of observed space already requires $k \gg 1$. And a horizon has been described only as an interface, not yet as a source of Hawking radiation.

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